PART I.

Containing

The PROPERTIES of BODIES,

Their LAWS of MOTION.

AND

The MECHANICAL POWERS.

The THIRD EDITION.

A

COMPENDIOUS SYSTEM

OF

Natural Philoſophy.

With NOTES

Containing the

MATHEMATICAL DEMONSTRATIONS,

AND

Some Occaſional REMARKS.

PART I.

The PROPERTIES of BODIES.
Their LAWS of MOTION. And
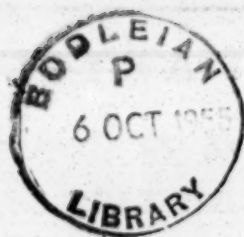The MECHANICAL POWERS.

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The THIRD EDITION.

A
COMPENDIOUS SYSTEM
OF
Natural Philofophy.

PART I.

The INTRODUCTION.

SO wild and extravagant have been the Notions of a great Part of Philofophers, both Ancient and Modern, that it is hard to determine, whether they have been more diftant in their Sentiments from Truth, or from one another; or have not exceeded the Fancies of the moft fabulous Writers, even Poets and Mythologifts. This was owing to a precipitate Proceeding in their fearching into Nature, their neglecting the ufe of Geometry and Experiment, the moft neceffary Helps to the finding out Caufes, and proportioning them to their Effects.

THE Manner of Philofophizing among the Ancients was to give Bodies certain *arbitrary*

Properties, such as best serv'd their Purpose in accounting for the Phænomena * of Nature; from whence proceeded so many various Sects of Philosophers; every one ascribing a different Cause to the same Appearance, as his particular Genius and Imagination led him.

THE chief Agreement observable among most of them, consists in this, *viz.* that they conceived all Bodies, as Compositions of Air, Earth, Fire, and Water, or some one or more of them, from whence they acquired the Name of Principles or Elements, which they still retain.

EPICURUS advanc'd a little farther, and asserted, that though Bodies consisted of some one or more of these, yet that they were not strictly Elements, but that these themselves consisted of Atoms; by an accidental Concourse of which, (as they were moving through infinite Space in Lines nearly parallel) all Things received their Form and Manner of Existence †.

DES CARTES has contrived an Hypothesis very different from the rest; he sets out with a

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* By a Phænomenon of Nature is meant any Motion or Situation of Bodies among one another, which offers itself to the Notice of our Senses, and is not the immediate Result of the Action of an intelligent Being.

† For the Opinions of the Ancient Philosophers consult *Diogenes Laertius* and *Stanley*'s Lives.

Suppofition that the Univerfe at firft was entirely full of Matter, that, from this Matter when firft put in Motion, there would neceffarily be rubbed off (by the grinding of the feveral Parts one againft another) fome Particles fufficiently fine to pafs through the hardeft and moft folid Bodies without meeting with any Refiftance : of thefe confifts his *Materia fubtilis,* or *Materia primi Elementi.* He imagined that from hence alfo would refult other Particles of a globular Form, to which he gave the name of *Materia fecundi Elementi.* Thofe which did not fo far lofe their firft Figure, as to come under the Denomination of *Materia primi* or *fecundi Elementi,* he called *Materia tertii Elementi;* and maintain'd that all the Variety, which appears in natural Bodies, was owing to different Combinations of thofe Elements.

He likewife fuppofes that God created a certain Quantity of Motion, and affigned it to this Mafs of Matter, and that That Motion (being once created) could no more be annihilated without an omnipotent Hand, than Body itfelf; in Confequence of which, he was obliged to teach, that the Quantity of Motion is always the fame : So that if all the Men and Animals in the World were moving, yet ftill there would be no more Motion, than when they were at Reft, the Motion which they had not, when at Reft, being

transferr'd

transferr'd to the Æther. So unaccountable are the Notions of this great Philosopher, that it is surprizing his Doctrine should have met with such universal Reception, and have got so strong a Party of Philosophers on its Side, that notwithstanding it was more absurd, than the Schoolmens *Substantial Forms*, they must all be exploded to make Way for his ingenious Hypothesis.

DES CARTES has been said by a late Writer *, to have joined to his great Genius an exquisite Skill in Mathematicks, and by mixing Geometry and Physics together, to have given the World Hopes of great Improvements in the latter. But this Writer ought to have considered that what he look'd upon in DES CARTES's Book of Principles, as Demonstrations, are only Illustrations, there not being a Demonstration from Geometry in all his Philosophical Works †.

THE present Method of Philosophizing establish'd by Sir ISAAC NEWTON, is to find out the Laws of Nature by Experiments and Observations, To this, with a proper Applica-

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* Mr. *Wotton* in his Reflections on Ancient and Modern Learning.

† See this Subject discuss'd more at large in *Keil*'s Introduction to his Examination of Dr. *Burnet*'s Theory.

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tion of Geometry, is owing the great Advantage the prefent Syftem of Philofophy has over all the preceeding ones, and the vaft Improvement it has received within the laft Age. It is indeed in vain to imagine, that a Syftem of Natural Philofophy can be framed by any other Method : for without Obfervations it is impoffible we fhould difcover the Phænomena of Nature, without Experiments we muft be ignorant of the mutual Actions of Bodies, and without Geometry we can never be certain whether the Caufes we affign be adequate to the Effects we would explain, as the various Syftems of Philofophy built on other Foundations, evidently fhew.

This Way of fearching into Nature was firft propofed by my Lord BACON *, profecuted by the *Royal Society*, the *Royal Academy* at *Paris*, the Honourable Mr. BOYLE, Sir ISAAC NEWTON, *&c.*

WHAT wonderful Advancement in the Knowledge of Nature may be made by this Method of Enquiry, when conducted by a Genius equal to the Work, will be beft underftood by confidering the Difcoveries of that excellent Philofopher laft mentioned. To Him it is principally owing, that we have now a

* See his *Novum Organum.*

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rational Syſtem of Natural Philoſophy; 'tis He who, by purſuing the ſure and unerring Method of Reaſoning from Experiment and Obſervation, joined with the moſt profound Skill in Geometry, has carried his Enquiries to the moſt minute and inviſible Parts of Matter, as well as to the largeſt and moſt remote Bodies in the Univerſe, and has eſtabliſh'd a Syſtem not ſubject to the Uncertainty of a mere Hypotheſis, but which ſtands upon the ſecure Baſis of Geometry itſelf.

C H A P.

CHAP I.

The Properties of Body.

IT being the Defign of *Phyfics* or Natural Philofophy to account for the Phænomena of the Material World, it is neceffary to begin with laying down the known Properties of Body.

THESE are 1. Solidity. 2. Extenfion. 3. Divifibility. 4. A Capacity of being moved from Place to Place. 5. A Paffivenefs or Inactivity. Which are the effential Properties of Body ; as appears from what follows.

1. SOLIDITY, called alfo Impenetrability, is that Power which Body has of excluding all others out of its Place.

THAT Body, as fuch, muft be endued with this Property follows from its Nature, for otherwife two Bodies might exift in the fame Place, which is abfurd. The fofteft are equally folid with the hardeft, for we find by Experiment, that the Sides of a Bladder, filled with Air or Water, can by no means be made to come clofe together *.

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* At *Florence* a hollow Globe of Gold was fill'd with Water, and then exactly clos'd ; the Globe thus clos'd was put into a Prefs driven by the Force of Screws ; the Water, finding no Room for a nearer Approach of its Particles toward each other, made its Way through the Pores of that clofe
<div align="right">Metal,</div>

2. THAT Body is extended, is felf evident, it being impoffible to conceive any Body, which has not Length, Breadth, and Thickneſs, that is, Extenſion.

3. IT is no leſs evident, that Body is diviſible, for ſince no two Particles of Matter can exiſt in the fame Place, it follows that they are really diſtinct from each other, which is all that is meant by being diviſible.

IN this Senſe the leaſt conceivable Particle muſt ſtill be diviſible, ſince it will conſiſt of Parts, which will be really diſtinct *. To illuſtrate this by a familiar Inſtance : Let the leaſt imaginable Piece of Matter be conceived lying on a ſmooth plain Surface, 'tis evident the Surface will not touch it every where, thoſe Parts therefore, which it does not touch, may be ſuppoſed ſeparable from the other, and ſo on as far as we pleaſe ; and this is all that

Metal, ſtanding in Drops like Dew on the Outſide, before the Globe would yield to the violent Preſſure of the Engine. V. *Acad. del Ciment.*

* This Propoſition is demonſtrated Geometrically thus, ſuppoſe the Line *A D* (*Fig.* 1.) perpendicular to *B F* and another as *G H* at a ſmall Diſtance from it alſo perpendicular to the ſame Line ; with the Centers *CCC*, &c. deſcribe Circles cutting the Line *G H* in the Points *e, e, e,* &c. Now the greater the Radius *A C* is, the leſs is the Part *e H*. But the Radius may be augmented in infinitum, and therefore the Part *e H* may be diminiſhed in the ſame Manner ; and yet it can never be reduc'd to nothing, becauſe the Circle can never coincide with the right Line *AF*; conſequently the Parts of any Magnitude repreſented by *G H* may be diminiſhed in infinitum. *Q. E. D.* V. *Keil's* Introd. ad Phyſ. Præl. 3, 4, 5, *Gravefande's* Elem. Math. Phyſ. L. 1. c. 4. Schol.

is

is meant, when we fay Matter is infinitely divifible.

How far Matter may actually be divided, may in fome manner be conceiv'd from hence* that a Piece of Wire, gilt with fo fmall a Quantity as eight Grains of Gold, may be drawn out to the Length of thirteen Thoufand Feet, the whole Surface of it ftill remaining cover'd with Gold †.

A Quantity of Vitriol, being diffolved and mix'd with nine Thoufand Times as much Water, will tinge the whole, confequently the Vitriol will be divided into as many Parts as there are vifible Portions of Matter in that Quantity of Water ‡.

THERE are Perfumes, which, without a fenfible Diminution of their Quantity, fhall fill a very large Space with their odoriferous Particles, which muft therefore be of an inconceivable Smallnefs, fince there will be a fuffici-

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* We have a furprizing Inftance of the Minutenefs of fome Parts of Matter, from the Nature of Light and Vifion. Let a Candle be lighted and placed in an open Plane, it will then be vifible two Miles round, confequently was it placed two Miles above the Surface of the Earth, it would fill with luminous Particles a Sphere, whofe Diameter was four Miles, and that before it had loft any fenfible Part of its Weight. The Force of this Argument will appear better when the Reader is acquainted with the Caufe of Vifion.

† *Keil*'s Introd. ad Phyf. Præl. 5. Religious Philof. Contempl. 25.

‡ Mem. de l'Acad. 1706.

B   ent

ent Number in every Part of that Space, senfi-
bly to affect the Organ of Smelling.

4. THAT all Matter is moveable, follows
from its being finite ; and to suppose it posi-
tively Infinite is absurd, because it consists of
Parts *.

5. BY the Passiveness or Inactivity of Mat-
ter, (commonly call'd its *Vis Inertiæ*) is meant
the Propensity it has to continue its State of
Motion or Reft, till some external Force acts
upon it. This will be farther explain'd under
the first Law of Nature.

CHAP. II.

Of Vacuum.

I. PLACE void of Matter is call'd empty
Space or *Vacuum.*

II. IT has been the Opinion of some Phi-
losophers, particularly the *Cartesians,* that Na-
ture admits not a *Vacuum,* but that the U-
niverse is entirely full of Matter : in conse-
quence of which Opinion they were oblig'd
to affert, that if every Thing contain'd in a
Veffel could be taken out or annihilated, the
Sides of that Veffel, however ftrong, would
come together ; but this is contrary to Expe-

* See Mr. *Law*'s Tranflation of ABp. *King de Origine
Mali.* Note 3.

rience,

rience, for the greateſt Part ᵃ of the Air may be drawn out of a Veſſel by means of the Air-Pump, notwithſtanding which it will remain whole, if its Sides are ſtrong enough to ſupport the Weight of the incumbent Atmoſphere.

III. Sʜᴏᴜʟᴅ it be objeᶜted here, that as it is impoſſible to extraᶜt all the Air out of a Veſſel, and that there will not be a *Vacuum* on that Account ; the Anſwer is, that ſince a very great Part of the Air, that was in the Veſſel, may be drawn out, as appears by the more quick Deſcent of light Bodies in a Receiver * when exhauſted of its Air, there muſt be ſome Vacuities between the Parts of the remaining Air : which is ſufficient to conſtitute a *Vacuum.* Indeed to this it may be objeᶜted by a *Carteſian,* that thoſe Vacuities are fill'd with *Materia ſubtilis,* that paſſes freely through the Sides of the Veſſel, and gives no Reſiſtance to the falling Bodies : but as the Exiſtence of this *Materia ſubtilis* can never be prov'd, we are not oblig'd to allow the Objeᶜtion; eſpecially ſince Sir Isᴀᴀᴄ Nᴇᴡᴛᴏɴ has found, that all Matter affords a Reſiſtance nearly in Proportion to its Denſity †.

Tʜᴇʀᴇ are many other Arguments to prove this, particularly the Motions of the Comets

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ᵃ A Veſſel cannot be entirely exhauſted of its Air, becauſe the Aᶜtion of the Pump depends on the Spring of that which remains in the Veſſel.

* By this Term is meant any Veſſel, out of which we extraᶜt the Air by the Air-Pump.

† *Newt.* Principia Lib. 2. Prop. 31. & 40. & Opt. Edit. 2. Book 3. Quer. 18, 19, 20, 21. *Deſagul.* Leᶜt. 1. Ann. 2.

through

through the Heavenly Regions without any fen-
fible Refiftance *; the different Weight of Bo-
dies of the fame Bulk &c. but thofe, being not
yet explain'd, are not fo proper to be infifted
on in this Place.

C H A P. III.

Of Attraction and Repulsion.

I. BESIDES the forementioned Proper-
ties of Matter, it has alfo certain
Powers or active Principles, known by the
Names of *Attraction* and *Repulfion*, probably
not effential or neceffary to its Exiftence, but
impreffed upon it by the Author of its Being,
for the better Performance of the Offices for
which it was defign'd.

II. ATTRACTION is of two Kinds. 1. Co-
hefion, or that by which minute Bodies, (or
the feveral Particles of the fame Body) when
placed afunder at very fmall Diftances, mutu-
ally approach each other; and then adhere or
ftick together, as if they were but one. 2. Gra-
vitation, or that by which diftant Bodies act
upon each other.

III. THE Attraction of Cohefion is prov'd
from abundance of Experiments, of which fome
of the moft obvious are as follows.

* *Defagul.* Lect. 1. Annot. 8.

1. LET

1. LET a small glass Tube (commonly call'd a Capillary Tube) open at both Ends, be dipt into a Vessel of Water, the Water will immediately rise up in the Tube to a certain Height above the Level. This Rise of the Water is manifestly owing to the Attraction of those Particles of the Glass, which lie in the inner Surface of the Tube immediately above the Water : Accordingly the Quantity of Water raised is always proportionable to the Largeness of that Surface *.

2. LET two Spheres of Quicksilver be placed near each other, and they will immediately run together, and form one Globule.

IV. THE Laws of this Attraction are 1*st*. That it acts only upon Contact, or at very small Distances ; for the Spheres, mentioned in the last Experiment, will not approach each other, till they are plac'd very near. 2. It

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* The Heights the Water rises to in different Tubes, are observ'd to be reciprocally as the Diameters of the Tubes, from whence it follows that the Quantities raised are as the Surfaces which raise them.

Dem. Let there be two Tubes, the Diameter of the first double to that of the second, the Water will rise half as high in the first as in the second, now was it to rise equally high in both, the Quantity in the first would be four times as great as in the second. (Cylinders of equal Heights being as the Squares of their Diameters ; 11. *El.* 14.) therefore since it is found to rise but half as high, the Quantity is but twice as much, and therefore as the Diameter; but the Surfaces of Cylinders are as their Diameters, therefore the Quantities of Water raised are also as the Surfaces. *Q. E. D.*
See the Dissertation on this Subject.   Part II.

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acts according to the Breadth of the Surfaces of the attracting Bodies, and not according to their Quantities of Matter. For, let there be two polish'd Glaſs Plates laid one upon another, in ſuch a Manner, as to touch at one End, and there make a very ſmall Angle : If two unequal Drops of Oil be put between theſe Plates, at equal Diſtances from the Line of Contact, ſo that the leaſt may touch both Glaſſes, they will then both move towards the Ends that touch, becauſe the Attraction of the Surfaces inclines that Way ; but the largeſt, touching the Glaſſes in moſt Points, will move the faſteſt. 3. 'Tis obſerv'd to decreaſe much more than as the Squares of the Diſtances of the attracting Bodies from each other increaſe : That is, whatever the Force of Attraction is at a given Diſtance, at twice that Diſtance it ſhall be more than four Times leſs than before *.

V. From hence it is eaſy to account for the different Degrees of Hardneſs in Bodies ; thoſe whoſe conſtituent Particles are flat or ſquare, and ſo ſituated as to touch in many Points, will be hard ; thoſe Particles which are more round, and touch in fewer Points, will conſtitute a ſofter Body ; thoſe which are ſpherical, or nearly of that Figure, will form a Fluid †.

* V. *Keilii* Opera Ed. 4to. p. 626.
† See *Rohault* in the Notes. pag. 105, 108, See Part II. Chap. I. §. 2. in the Notes. *Newtoni.* Optic. p. 335.

VI. At-

VI. ATTRACTION of Gravitation is that, by which diftant Bodies act upon each other. Of this we have daily Inftances in the falling of heavy Bodies toward the Earth.

VII. THE Laws of this Attraction are 1. That it decreafes, as the Squares of the Diftances between the Centers of the attracting Bodies increafe. Thus, a Body which at the Surface of the Earth *(i. e.* about the Diftance of four Thousand Miles from its Center) weighs ten Pounds, if it was plac'd four Thousand Miles above the Surface of the Earth, *i. e.* twice as far diftant from the Center as before, would weigh four Times less; if thrice as far, nine Times less, *&c.* The Truth of this Propofition is not to be had from Experiments, (the utmoft Diftance we can convey Bodies to, from the Surface of the Earth, bearing no Proportion to their Diftance from its Center,) but is fufficiently clear from the Motions obferv'd by the Heavenly Bodies. 2. Bodies attract one another with Forces proportionable to the Quantities of Matter they contain; for all Bodies are obferv'd to fall equally faft in the exhaufted Receiver, where they meet with no Refiftance. From whence it follows, that the Action of the Earth upon Bodies is exactly in Proportion to the Quantities of Matter they contain; for was it to act as ftrongly upon a less Body as upon a larger, the leaft Body, being moft eafily put into Motion,

<div align="right">would</div>

would move the faſteſt. Accordingly, it is obſervable, that the Weight of a Body is the ſame, whether it be whole, or ground to Powder *.

VIII. From hence it follows, that, was a Body to deſcend from the Surface toward the Center of the Earth, it would continually become lighter and lighter, the Parts above attracting it, as well as thoſe below; in which Caſe it is demonſtrated by Mathematicians, that the Gravity would decreaſe with the Diſtance of the Body from the Center †.

Scholium.

* *Gravefande* Lib. 4. Chap. 11. *Cotes*'s Preface to *Newton*'s Princip.

† *Dem.* Let there be a Body as *P*, (*Fig.* 2.) placed any where within a concave Sphere, as *AB*, which let us ſuppoſe divided into an infinite Number of thin concentric Surfaces; I ſay, the Body P will be attracted equally each way by any one of theſe *v. g.* the interior *HIKLM.* Let there be Lines as *IL*, *HK*, &c. drawn through any Point of the Body *P*, in ſuch a manner as to form the Surface of two ſimilar Figures, ſuppoſe Cones, the Diameters of whoſe Baſes may be *IH*, *KL*, which let be infinitely ſmall. Theſe Baſes (being as the Squares of the Lines *IH, KL*) (2. *Elem.* 12.) will be directly, as the Squares of their Diſtances from *P* (for the Triangles *IPH, KPL*, being infinitely ſmall, are ſimilar.) But thoſe Baſes include all the Particles of Matter in the interior Surface, that are oppoſite to each other; the oppoſite Attractions are therefore in the ſame Ratio with thoſe Baſes, that is as the Squares of the Diſtances *PK, PI.* But the Attraction is inverſely, as the Squares of the Diſtances of the attracting Bodies, §. 7. *i. e.* inverſely as the Squares of the ſame Diſtances *PK, PI*; theſe two Ratios therefore deſtroying each other, it is evident, that if the Concavity of the Sphere was filled with Matter, that alone, which lies nearer the Center than the Body can effect it, the reſpective Actions of all the Parts, that are more diſtant, being equal,

and

Scholium. It may be proper to obferve here, that when Philofophers fpeak of Bodies gravitating to, or attracting each other, that Body is faid to gravitate to another, which moves towards it, while the other actually is, or appears to be, at reft, and this other is, faid to attract the former; though indeed the Force being mutual and equal on both Sides (as will be explain'd under the third Law of Nature) the fame Term might be apply'd to either the gravitating or attracting Body.

It is farther to be obferv'd, that when we ufe the Terms, Attraction or Gravitation, we do not thereby determine the Phyfical Caufe of it, as if it proceeded from fome fuppofed *occult* Quality in Bodies; but only ufe thofe Terms to fignify an Effect, the Caufe of which lies out of the reach of our Philofophy. Thus, we may fay, that the Earth attracts heavy Bodies; or that fuch Bodies tend or gravitate to the Earth : although at the fame time we

and in contrary Directions, fince the fame is demonftrable of any of the remaining concentric Surfaces. Let us fee then what Effect that, which lies nearer the Center than the Body, will have upon it, which may be confidered as a Sphere, on whofe Surface the Body is plac'd. The Diftances of each Particle of Matter from the Body, (taken collectively) will be as the Diameter of the Sphere, or as the Radius, *i. e.* as the Diftance of the Body from the Center : their Action therefore upon the Body will be inverfely as the Square of that Diftance : but the Quantity of Matter will be as the Cube of that Diftance; (18. *Elem.* 12.) the Attraction therefore will be alfo in that Proportion. Now, thefe two Ratios being compounded, the Attraction will be only as the Diftance of the Body from the Center. *2. E. D.*

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are wholly ignorant: whether this is effected by fome Power, actually exifting in the Earth, or in the Bodies, or external to both ; fince it is impoffible any Error in our Reafonings can follow from hence : it being evident, that all the Confequences of fuch Tendency muft be the fame, let the Caufe be where, or what it will.

X. REPULSION is that Property in Bodies, whereby, if they are placed juft beyond the Sphere of each others Attraction of Cohefion, they mutually fly from each other.

THUS, if an oily Subftance, lighter than Water, be placed on the Surface thereof, or if a Piece of Iron be laid on Mercury, the Surface of the Fluid will be deprefs'd about the Body laid on it : This Depreffion is manifeftly occafion'd by a repelling Power in the Bodies, which hinders the Approach of the Fluid towards them.

BUT it is poffible in fome Cafes to prefs or force the repelling Bodies into the Sphere of one anothers Attraction ; and then they will mutually tend towards each other ; as when we mix Oyl and Water till they incorporate *.

XI. BESIDES the general Powers forementioned, there are fome Bodies, that are endued with another, call'd *Electricity.* Thus Amber,

* We have an undeniable Proof of this Repulfive Force in Sir *Ifaac Newton*'s Opticks. B. 3. and Query 31.

Jet,

Jet, Sealing-Wax, Agate, Glafs and moft Kinds of Precious Stones attract and repel light Bodies at confiderable Diftances.

THE chief Things obfervable in thefe Bodies are. 1. That they don't act, but when heated. 2. That they act more forcibly when heated by rubbing, than by Fire. 3. That, when they are well heated by rubbing, light Bodies will be alternately attracted and repell'd by them, but without any obfervable Regularity whatever. 4. If a Line of feveral Yards in length has a Ball, or other Body fufpended at one End, and the other End be fixed to a glafs Tube; when the Tube is heated by rubbing, the Electrical Virtue of the Glafs will be communicated from the Tube to the Ball, which will attract and repel light Bodies in the fame Manner, as the Glafs itfelf does. 5. If the glafs Tube be emptied of Air, it lofes its Electricity *.

XII. LASTLY, the Loadftone is obferv'd to have Properties peculiar to itfelf, as that by which it attracts and repels Iron, the Power it communicates to the Needle, and feveral others †.

* See *Hauksbee*'s Experiments. *Philofoph. Tranfact.* No. 326.

† Several Solutions of thefe Properties of *Electricity* and *Magnetifm* have been attempted by different Philofophers, but all of them fo unfatisfactory as not to deferve a particular Account in this Place. See *Chamber*'s Dictionary in *Electricity*, and *Des Cartes* Opera Philofophica. P. IV. §. 133, with feveral others referr'd to in *Quæftiones Philofophicæ*. *Defagul.* Lect. I. §. 33.

C 2   CHAP.

C H A P. IV.

Of the Laws of Motion, commonly cal-
led Sir ISAAC NEWTON'*s Laws of*
Nature.

I. **A**LL Bodies continue their State of Reft, or uniform Motion in a right Line, till they are made to change that State by fome external Force impreffed upon them.

THIS Law is no other, than that univerfal Property of Bodies, called Paffivenefs or Inactivity; whereby they endeavour to continue the State they are in, whatever it be. Thus a Top only ceafes to run round on Account of the Refiftance it meets with from the Air, and the Friction of the Plane whereon it moves. And a Pendulum, when left to vibrate in *vacuo,* where there is nothing to ftop it, but the Friction, arifing from the Motion of the Pin on which it is fufpended, continues to move much longer, than one in the open Air.

II. THE Change of Motion, produc'd in any Body, is always proportionable to the Force, whereby it is effected; and in the fame Direction, wherein that Force acts.

THIS is an immediate Confequence of this Axiom, the Effect is always proportionable to

its

its Caufe. For Inftance, if a certain Force produces a certain Motion, a double Force will produce double the Motion ; a triple Force triple the Motion, &c. If a Body is in Motion, and has a new Force impreffed on it in the Direction wherein it moves, it will receive an Addition to its Motion, proportional to the Force impreffed ; but if the Force acts directly contrary to its Motion, the Body will then lofe a proportional Part of its Motion : Again, if the Force is impreffed obliquely, it will produce a new Direction in the Motion of the Body, more or lefs different from the former in Proportion to its Quantity and Direction *.

III. Re-

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* This Cafe is expreffed more accurately by Mathematicians thus. If the Proportion and Direction of two Forces, acting upon a Body at the fame Time, be reprefented by the Sides of a Parallelogram, the Diagonal of that Parallelogram will reprefent the Proportion and Direction of their united Forces.

Dem. Let the Body *A (Fig.* 3.) be impell'd with a Force, which would carry it to *E,* in the fame Time that another, acting upon it in the Direction *AD,* would carry it to *D.* Imagine that while the Body paffes to *E,* the Line *AD* (in which the Body moves by the other Force) moves to *EB,* in a Direction parallel to itfelf; when the Body has advanc'd to *G* in the Line *AE,* the Line *AD* will have got to *GF,* and the Body will have paffed over *GH.* fuch a Part of it, as bears the fame Proportion to the whole Line *GF,* as *AG* does to *AE,* that is, *GH* (the fhorter Side of the Parallelogram *GM,)* is to *GF,* or, which is the fame Thing, to *EB* (the fhorter Side of the Parallelogram *ED,* as *AG* (the longer Side of the former) is to *AE* the longer Side of the latter.) from whence the Parallelograms are fimilar, *El.* 6. *Def.* 1. and confequently, by 24. *El.* 6. the Point *H* is in the Diagonal, that is, the Body will always be found in the Line *AB.* Q. E. D.

Coroll.

III. Reaction is always contrary, and equal to Action; or the Actions of two Bodies upon each other, are equal, and in contrary Directions.

Thus, suppose a Stone, or other Load to be drawn by an Horse; the Load reacts upon the Horse, as much as the Horse acts upon the Load; for, the Harness, which is stretch'd equally between them both Ways, draws the Horse towards the Stone, as much as it does the Stone towards the Horse; and the progressive Motion of the Horse is as much retarded by the Load, as the Motion of the Load is promoted by the Endeavour of the Horse *. This will be better explain'd from the following Instance; let a Person, sitting in a Boat,

Coroll. From hence we have an easy Method of resolving a given Motion into any two, or more Directions whatever; *viz.* by describing a Parallelogram about the given Direction as a Diagonal, the two Sides of which will represent the Directions sought. Thus, suppose a Body was impell'd in the Line *AB*, we may conceive it as acted upon by two Forces at the same Time, one towards *E*, the other towards *D*, or any other two whatever, provided the Lines be drawn of such length, that, when the Parallelogram is compleated, the given Line *AB* shall be its Diagonal.

* It may be thought perhaps, that (two equal and contrary Forces destroying one another) the Horse will in this Case not be able to move at all, because the Load draws him back, as much as he draws the Load forwards. But it is to be observ'd that the Strength of the Horse is not properly exerted upon the Load but upon the Ground; consequently the Ground, reacting and continuing at Rest, pushes the Horse forward with just so much Force as the Horse exerts, above what is counteracted by the Load.

draw

draw another Boat equally heavy towards him, they will both move towards each other with equal Velocities: Let the Boat he fits in be the lighteft, and it will move the fafteft; becaufe the Action being equal on both Sides, the fame Quantity of Motion will be given to each Boat, that is, the leffer will have the greater Velocity *.

WE have a farther Confirmation of this from Attraction. Suppofe two Bodies attracting one another, but prevented from coming clofe together by fome other Body placed between them: If their refpective Actions, by which they tend towards each other, were not equal on both Sides, then would the intermediate Body be preffed more one Way than the other, and fo all three would of themfelves begin to move; but that three Bodies fhould be put into Motion after this Manner, when no external Force acts upon them, is contrary to Experience, confequently whatever different Degrees of Force, any two Bodies may be capable of exerting, their mutual Actions on each other are always equal. This may be try'd with a Loadftone and Iron; which, being put into proper Veffels, contiguous to one another, and made to float on the Surface of Water, will be an exact Counterbalance to

* See the Diftinction between Motion and Velocity. Chap. 9.

each

each other, and remain at Reſt, whatever be the attractive Power of the Loadſtone, or the Proportion of their reſpective Magnitudes.

THESE Laws receive an abundant Additional Proof from hence, *viz.* that all the Concluſions that are drawn from them, in Relation to the Phænomena of Bodies, how complicated ſoever their Motions be, are always found to agree perfectly with Obſervation. The Truth of which ſufficiently appears in all Parts of the *Newtonian* Philoſophy *.

CHAP. V.

The Phænomena of Falling Bodies.

I. THE Laws of Nature being thus explained, we proceed to account for ſome of thoſe Phænomena, which are ſolvable by them.

II. To begin with thoſe of Falling Bodies. Conſtant Experience ſhews, that Bodies have a Tendency towards the Earth, which is call'd Gravity, the Laws of which were enumerated in Chap. 3. §. 7.

III. THE Height, Bodies can be let fall from, bears ſo ſmall a Proportion to their Diſtance from the Center of the Earth, that it cannot

* See theſe Laws explain'd more at large by *Cheyne* in his Principles of Philoſophy. *Keil's* Introd. ad Phyſ. Præl. 11, 12.

ſenſibly

senfibly alter their Gravity; which therefore may be conceiv'd, as acting conftantly and uniformly upon them, during the whole Time of their Fall: From whence they muft neceffarily acquire, at every Inftant, an equal Degree of Velocity, which on that Account will conftantly increafe, in Proportion to the Time the Body takes up in falling.

IV. THE Spaces Bodies fall through in different Times, reckoning from the Beginning of their Fall, are as the Squares of thofe Times; thus, a Body will fall four Times as far in two Minutes, as it does in one; and nine Times as far in three, fixteen Times as far in four, &c. *

V. FROM

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* In order to demonftrate this Propofition, it will be neceffary to lay down the following Theorem, *viz.*

That the Space a Body paffes over, with an uniform Motion, is in a Ratio compounded of the Time and Velocity. For the longer a Body continues to move uniformly, the more Space it moves over; and the fafter it moves during any Interval of Time, the farther it goes; therefore the Space is in a Ratio compounded of both, that is, is had by multiplying one into the other.

Coroll. Therefore the Area of a Rectangle, one of whofe Sides reprefents the Celerity a Body moves with, and the other the Time of its Motion, will exprefs the Space it moves through.

This being premifed, let the Line *AB* (*Fig.* 4.) - reprefent the Time a Body takes up in falling, and let *BC* exprefs the Celerity acquir'd by its Fall; farther let the Line *AB* be divided into an indefinite Number of fmall Portions, *ei, im, mp,* &c. and let *ef, ik, mn, pq,* &c. be drawn parallel to the Bafe. Now it is evident from §. 3. (*viz.* that the Velocities are as the Times in which they are acquir'd) that the Lines *ef, ik, mn, pq,* &c. being to each other (4. *El.* 6.) as the Lines *Ae, Ai, Am, Ap,* &c. will reprefent the Celerities in the Times reprefented by thefe: that is, *ef* will be as the Velocity of the

D   Body

V. FROM this Propofition it follows, that a Body falls three Times as far, in the fecond Portion of Time, as it does in the firft; five Times as far in the third; feven Times in the fourth, and fo on in the Series of the odd Numbers: For otherwife it could not fall four Spaces in two Minutes, and nine in three, as the Propofition afferts *.

VI. THE

Body in the fmall Portion of Time *ei*, and *ik* will be as the Velocity in the Portion of Time *im*; in like Manner *pq* will be as the Velocity in the Portion of Time *po*, which Portions of Time being taken infinitely fmall, the Velocity of the Body may be fuppos'd the fame, during any whole Portion: and confequently, by the Corollary of the foregoing Theorem, the Space run over in the Time *ei* with the Velocity *ef* may be reprefented by the Rectangle *if*: in like Manner the Space run over in the Time *im*, with the Celerity *ik*, may be exprefs'd by the Rectangle *mk*; and that run over with the Celerity *mn* in the Time *mp*, by the Rectangle *pn*; and fo of the reft. Therefore the Space run over in all thofe Times will be reprefented by the Sum of all the Rectangles, that is, by the Triangle *ABC*, for thofe little triangular Deficiencies, at the End of each Rectangle, would have vanifhed, had the Lines *ei*, *im*, *mp*, &c. been infinitely fhort, as the Times they were fuppofed to reprefent. Now as the Space, the Body defcribes in the Time *AB*, is reprefented by the Triangle *ABC*, for the fame Reafon the Space pafs'd over in the Time *Ao* may be reprefented by the Triangle *Aor*, but thefe Triangles, being fimilar, are to each other, as the Squares of their homologous Sides *AB* and *Ao* (20. *El.* 6.): that is, the Spaces reprefented by the Triangles are to each other, as the Squares of the Times reprefented by the Sides. *Q. E. D.*

* This may alfo be fhewn in the following Manner. Let the Triangle *ABC* (*Fig.* 4.) be divided into leffer ones, as in *Fig.* 5. each equal to *Dbr*, which reprefents the Space defcribed by the falling Body in *Db* the firft Portion of Time; 'tis evident that, in *bc* the fecond Portion of Time, there are three fuch Triangles defcribed, *viz.* thofe that lie between the

Lines

VI. The Spaces, defcrib'd by falling Bodies in different Times, are as the Squares of the laft acquir'd Velocities. For by §. 4. the Spaces are as the Squares of the Times, and by §. 3. the Velocities are as the Times; therefore the Spaces are alfo as the Squares of the Velocities.

VII. The Space a Body paffes over, from the Beginning of its Fall in any determinate Time, is half what it would defcribe in the fame Time moving uniformly with its laft acquir'd Velocity *.

VIII. In like Manner, when Bodies are thrown up perpendicularly, their Velocities decreafe, as the Times they afcend in increafe; their Gravity deftroying an equal Portion of their Velocity every Inftant of their Afcent.

IX. The Heights Bodies rife to, when thrown perpendicularly upwards, are as the Squares of the Times fpent from their firft fetting out, to the Moment they ceafe to rife. That is, if a Body is thrown with fuch a Degree of Velocity, as to continue rifing twice as

Lines *br* and *cs*; in *cd* the third Portion of Time, five fuch, *viz.* all between *cs* and *dt*; in *df* the next equal Portion of Time, feven fuch, *&c.*

* For let the Time be *AB*, (*Fig.* 4.) and the laft Velocity *BC*, the Space the Body runs over, while it is acquiring that Velocity, is as *ABC*, but the Space it would pafs over in the Time *AB*, was it to move uniformly with the Celerity *BC*, is by the Theorem (Note p. 25.) as the Space *ABCD*, double the former. *Q. E. D.*

long

long as another, it shall ascend four Times as high; if thrice, nine Times as high, &c.

THESE two are the converse of the third and fourth Sections *.

CHAP. VI.

Of the Descent of Bodies on oblique Planes, and of Pendulums.

WHEN a Body descends on an oblique Plane, its Motion is continually accelerated by the Action of Gravity, but in a less Degree, than when it descends perpendicularly; its free Descent in this Case being hinder'd by the Interposition of the Plane: From whence it follows, that what was said in the last Chapter, concerning the perpendicular Descent of Bodies, is true of such as fall on oblique Planes, Allowance being made for the Difference of Acceleration.

II. THE Effect Gravity has upon a Body falling down an oblique Plane, is to that which it exerts upon another falling freely, as the perpendicular Height of the Plane is to its Length †.

III. THE

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* See *Keil's* Introd. ad Phyf. Præl. 11. *Gravefande* L. 1. Ch. 17.

† *Dem.* Let *AC* (*Fig.* 6.) be the inclin'd Plane, the Body at *A*, and the Action of Gravity, whereby it endeavours to fall

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III. The Space, through which a Body falls down the oblique Side of a Plane, is to that through which it would fall perpendicularly in the fame Time, as the perpendicular Height of the Plane is to its Length *.

For the Space, a Body falls through in any determinate Time, whether down an inclined Plane, or not, is as the Effect of the Gravity with which it is acted upon during that Time; but the Gravity, with which a Body defcends down the oblique Side of a Plane (by the laft Propofition) is to that with which it falls perpendicularly, as the perpendicular Height of the Plane to its Length: The Space therefore, which a Body falls through obliquely, is to that which it would pafs through perpendicularly in the fame Time, alfo in that Proportion.

perpendicularly, reprefented by the Line AB; let AD be perpendicular to AC, AD will then reprefent the Direction by which the Plane acts upon the Body (for all Bodies act in Lines perpendicular to their Surfaces) let then thofe two Forces be refolved into one in the Direction AC, (as fhewn in Note to §. 4. Chap. IV.) by compleating the Parallelogram BD, whofe Diagonal will be AG. In order to this BG muft be let fall perpendicularly upon AC (that it may be parallel to the oppofite Side of the Parallelogram AD) confequently (8. *Elem.* 6.) AG is to AB as AB to AC, that is, the Tendency of the Body down the Plane is to its perpendicular Tendency, as AB is to AC. Q. E. D.

* From this Propofition it follows, that fuppofing BG (*Fig.* 6.) perpendicular to AC) the Body would fall from A to G, in the fame Time another would fall from thence to B; for, as was obferv'd (Note the laft) AG is to AB, as AB to AC.

IV. The

IV. The Velocity, a Body acquires by falling perpendicularly, is to that, which it acquires by falling obliquely in the fame Time, as the Space of its perpendicular Defcent is to that of its oblique one *.

V. The Time, in which a Body defcends through the oblique Side of a Plane, is to that in which it falls through the perpendicular Height of the fame, as the Length of the oblique Side is to its Height †.

VI. A Body acquires the fame Velocity in falling down the oblique Side of a Plane, as

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* Since by the Note to Section the laft, a Body falls to G, (*Fig.* 6.) in the fame Time another falls to B, and by (Chap. V. §. 7.) the Space, a falling Body paffes over in any Time, is half that which it would run over in the fame Time moving uniformly with its laft acquir'd Velocity, it follows that the Body falling down the oblique Plane would pafs over double the Space AG, moving uniformly with its laft acquir'd Velocity, in a Portion of Time equal to that in which it was acquired ; likewife double the Space AB would be paffed over by the other Body, moving uniformly with its laft acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; but fince the Velocities of Bodies moving uniformly are as the Spaces they run over in equal Times, the Velocities of the Bodies in G and B are to each other as double the Lines AG and AB, that is, as the Lines themfelves, which by §. 3. are as the Spaces run through in the fame Time, from whence the Propofition is clear.

† *Dem.* The Square of the Time in which AC (*Fig.* 6.) is run over, is to the Square of the Time in which AG is run over, as AC to AG, (by Chap. V. §. 4.) that is, fince AC, AB, AG are continually proportional (8. *Elem.* 6.) as the Square of AC to the Square of AB (by *Def.* 10. *Elem.* 5.) therefore the Times themfelves are as the Lines AC and AB, that is, as the oblique Side of the Plane to the perpendicular Height. 2. *E. D.*

it would do, if it fell freely through the perpendicular Height of it *.

VII. A Body takes up the same Time in falling through the Chord of a Circle, whether it be long or short, as it does in falling perpendicularly through the Diameter of the same Circle †.

VIII. Upon this is founded the Theory of Pendulums: For from hence it follows, that supposing a Pendulum could be made to vibrate in a Chord of a Circle, instead of an Arch, all its Vibrations would require the same Time, whether they were large or small ‡.

IX. From hence we see the Reason, why the shorter Arches a Pendulum describes, the

* Dem. The Square of the Velocity, a Body acquires by falling to G, is to the Square of the Velocity it acquires by falling to C, as the Space AG to the Space AC (by Chap. V. §. 4.) that is (by 8. *Elem.* 6. and *Def.* 10. *Elem.* 5.) as AGq to ABq; consequently the simple Velocity at G is to that at C, as AG to AB: But since AG is run over in the same Time AB is (see Note to §. 3.) the Velocity in G is also to the Velocity in B, as AG to AB, (by §. 4.) and consequently since the Velocities both in C and B bear the same Proportion to that in G, they must be equal to each other. *Q. E. D.*

† Dem. It was demonstrated (§. 3.) that a Body will fall from A to G, *(Fig. 7.)* on the inclin'd Plane AC, in the same Time another would fall freely to B, provided AGB is a right Angle, in which Case AG (by 31. *Elem.* 3.) is a Chord of that Circle of which AB is the Diameter; therefore a Body falls through the Chord, &c. Q. E. D.

‡ This may be illustrated by conceiving the last Figure inverted (as in *Fig.* 8.) where supposing the Ball suspended in such a Manner, as to swing in the right Line GA instead of the Arch GA, it would always fall through it in the same Time, however long or short it was, for the Inclination of the Line GA to the horizontal Line BC, is not alter'd by inverting the Figure.

nearer its Vibrations come to an Equality, for small Arches differ less from their Chords than large ones. But if the Pendulum is made to vibrate in a Curve, which Mathematicians call a *Cycloid* ; each Swing will then be perform'd in the same Time, whether the Pendulum moves through a larger or lesser Space. For the Nature of this Curve is such, that the Tendency of a Pendulum towards the lowest Point of it, is always in Proportion to its Distance from thence ; and consequently let that Distance be more or less, it will always be run over by the Pendulum in the same Time *.

X. THE Time of the Descent and Ascent of a Pendulum, supposing it to vibrate in the Chord of a Circle, is equal to the Time in which

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* The Description of a *Cycloid*,

Upon the right Line *AB*, *(Fig. 9.)* let the Circle *CDE* be so plac'd, as to touch the Line in the Point *C*, then let this Circle roll along upon it from *C* to *H*, as a Wheel upon the Ground, then will the Point *C* in one Revolution of the Circle describe the Curve *CKH*, which is called a Cycloid. Now suppose two Plates of Metal bent into the Form *HK* and *KC*, and placed in the Situation *LH* and *LC*, in such Manner, that the Points *H* and *C* may be apply'd to *L*, and the Points answering to *K* be apply'd to *H* and *C*. This done, if a Pendulum as *LP*, in Length equal to *LH*, be made to vibrate between the Plates or Cheeks of the Cycloid *LC* and *LH*, it will swing in the Line *CKH*; and the Time of each Vibration, whether the Pendulum swings through a small or a great Part of the Cycloid, will be to the Time a a Body takes up in falling perpendicularly through a Space equal to *IK*, (half the Length of the Pendulum ;) as the Circumference of a Circle to its Diameter, and consequently it will always be the same.

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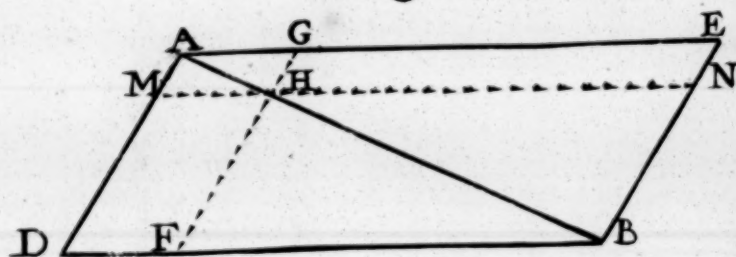*E* be
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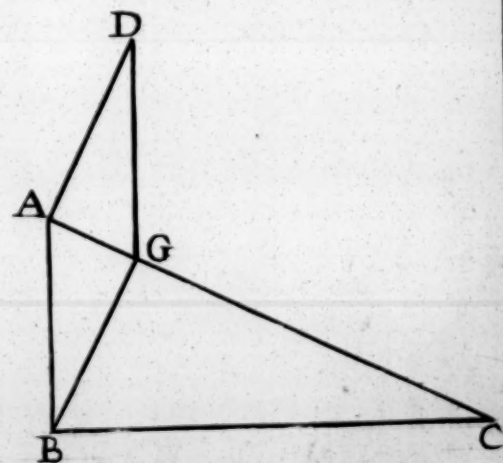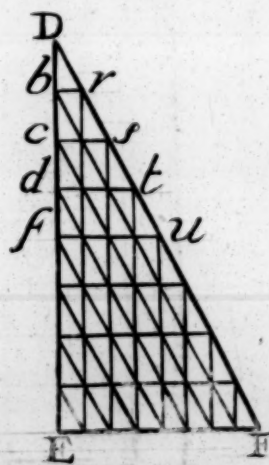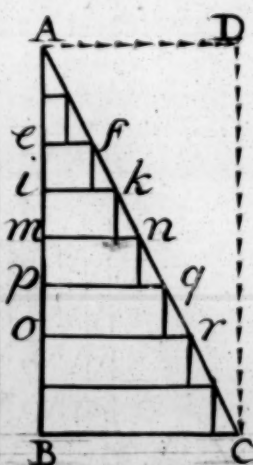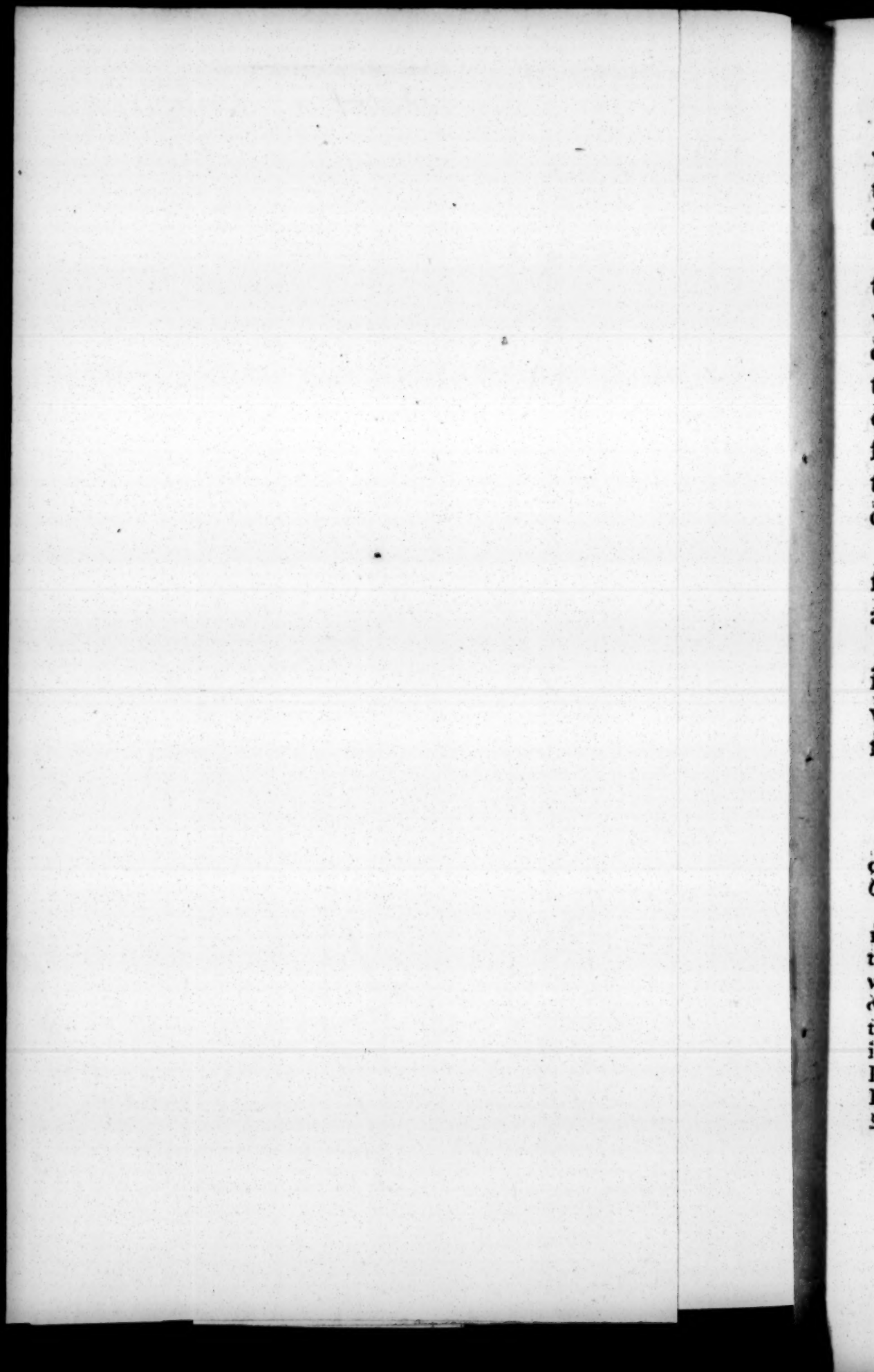be the

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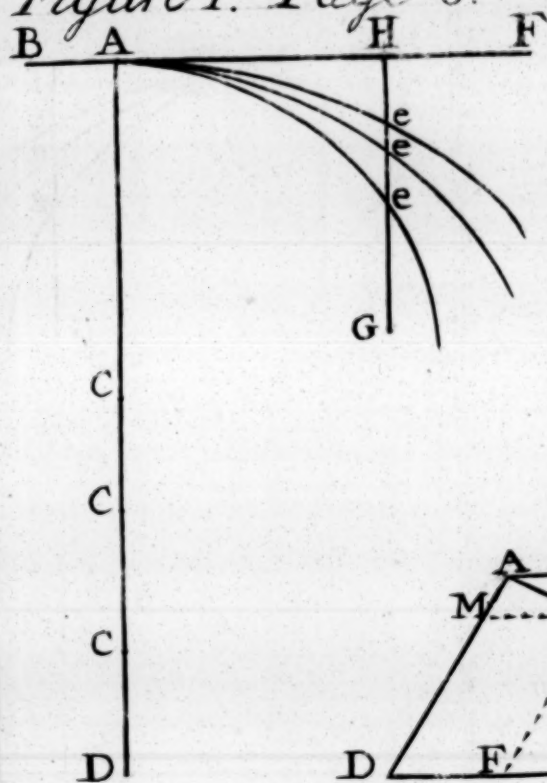
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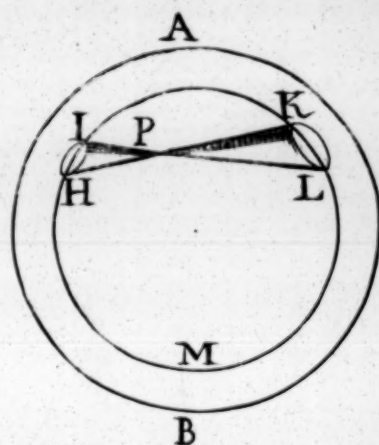
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F. 4. P. 25.   F. 5. P. 26.   F. 6. P. 28.

which a Body falling freely would defcend through eight Times the Length of the Pendulum.

For the Time of the Defcent alone upon the Chord is equal to that in which a Body would fall through the Diameter of the Circle (by §. 7); that is, twice the Length of the Pendulum : But in twice that Time (*viz.* during a whole Vibration) the Body would fall four Times as far (Chap. V. §. 4.), that is, through eight Times the Length of the Pendulum.

XI. The Times, that Pendulums of different Lengths perform their Vibrations in, are as the fquare Roots of their Lengths *.

XII. The Center of *Ofcillation* is a Point in which, if the whole Gravity of a Pendulum was collected, the Time of its Vibration would not be alter'd thereby †; this is the Point from whence

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They that would fee a Demonftration of this and feveral other Things relating to this Curve, may confult *Huygens Horol. Ofcillatorium*, or *Cotes*'s *Harmonia Menfurarum*.

* *Dem.* Let there be two Pendulums *A* and *B* (*Fig.* 10. and 11.) of different Lengths, the Time the firft vibrates in (fuppofe through a Chord) is equal to the Time in which a Body would fall freely through *DA*, the Diameter of the Circle (as demonftrated §. 7.); in like Manner the Time *B* vibrates in is that in which a Body would fall through *FB*. Now the Times in which Bodies fall through different Spaces are as the fquare Roots of thofe Spaces, that is, of *DA* and *FB*, or of their Halves *CA* and *CB*, *i. e.* of the Lengths of the Pendulums. Q. E. D.

† The Rule for finding the Center of Ofcillation.

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whence the Length of a Pendulum is measur'd, which in our Latitude, in a Pendulum that swings Seconds, is thirty nine Inches and two Tenths.

XIII. The Squares of the Times in which Pendulums, acted upon by different Degrees of Gravity, perform their Vibrations in, are to each other, inversly as the Gravities *.

From whence it follows, that a Pendulum will vibrate slower when nearer the Equator, than the same when nearer the Poles; for the

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If the Ball *AB (Fig. 12.)* be hung by the String *CD,* whose Weight is inconsiderable, the Center of Oscillation is found thus; suppose *E* the Center of the Globe, take the Line *K* of such a Length, that it shall bear the same Proportion to *ED* as *ED* to *EC,* then *EH* being made equal to $\frac{2}{3}$ of *K,* the Point *H* shall be the Center of Oscillation.

If the Weight of the Rod *CD* be too considerable to be neglected divide *CD (Fig. 13.)* in *I,* so that *DI* may be equal to $\frac{1}{3}$ of *CD,* and make a Line as *G,* in the same Proportion to *CI,* that the Weight of the Rod bears to that of the Globe, then having found *H* the Center of Oscillation of the Globe, as before, divide *IH* in *L,* so that *IL* may bear the same Proportion to *LH,* as the Line *CH* bears to the Line *G;* then will *L* be the Center of Oscillation of the whole Pendulum. See *Huygens* Horol. Oscillat.

* *Dem.* The Spaces, falling Bodies descend through, are as the Squares of the Times, when the Gravity by which they are impell'd is given (Chap. V. §. 4); and as the Gravity when the Time is given (for the Sum of the Velocities produced in any Time will always be as the generating Forces:) Consequently when neither is given, they are in a Ratio compounded of both; the Squares of the Times are therefore inversly as the Gravities. [*For if in* 3 *Quantities* a, b, c; a *is as* b c, *then* b : $\frac{a}{c}$, *i. e. if* a *is given, as* $\frac{1}{c}$ *or as* c *inversly.*] But if

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the Gravity of all Bodies is lefs, the nearer they are to the Equator; *viz.* on account of the fpheroidical Figure of the Earth, and its Rotation about its Axis, as will be explain'd hereafter. To which we may add the Increafe of the Length of the Pendulum occafion'd by the Heat in thofe Parts; (for we find by Experiment, that Bodies are inlarged in every Dimenfion, in Proportion to the Degree of Heat that is given them;) for which Reafon (§. 11.) the Vibrations of the Pendulum will alfo be flower.

CHAP. VII.

Of Projectiles.

A BODY, projected in a Direction parallel or oblique to the Horizon, would proceed on *in infinitum* in a right Line (by the firft Law of Nature); but being continually accelerated towards the Earth by its Gravity, it will defcribe a Curve called a *Parabola* *.

the Squares of the Times, in which Bodies fall through given Spaces, are inverfly as the Gravities by which they are acted upon; then the Squares of the Times, in which Pendulums of equal Lengths perform their Vibrations, will be alfo in the fame Ratio, on account of the conftant Equality between the Time of the Vibration of a Pendulum, and of the Defcent of a Body through eight Times its Length (§. 12.)

  * *Dem.* Let us fuppofe the Body thrown from *A* in the Direction *AB* horizontally *(Fig. 14.)* or obliquely *(Fig. 15.)* it would (if not attracted towards the Earth) move uniformly

from A towards B, that is, in equal Times it would defcribe equal Parts of the Line AB, as AC, CD, DE, &c. but, if in the firft Portion of Time, while it would move from A to C, it would have defcended from A to G by its Gravity, had it only been let drop from thence; it will, by a Compofition of thefe two Motions (Chap. IV. §. 2.) at the End of that Time be found in H, the oppofite Angle of the Parallelogram $ACGH$. Then in twice that Time, *viz.* while it would have moved over two equal Portions, or from A to D, it would fall downwards to M, four Times as far as before (Chap. V. §. 4) and will therefore be found in L, fuppofing DL equal and parallel to AM. Then again in three Portions of Time, or while it would have moved over three Divifions, that is, from A to E, it would have fallen downwards nine Times as far as in the firft Portion of Time, and therefore being carried by thefe two Motions will at the End of that Time be found in K, fuppofing EK, or its equal AN, nine Times as long as AG or CH, &c. Therefore the Lines CH, DI, EK, &c. which are to each other as the Numbers 1, 4, 9, &c. are as the Squares of the Lines AC, AD, AE; (thefe being only as the Numbers 1, 2, 3.) But this is the Property of the Parabolic Curve (See De L'*Hofpital* B. I. Prop 1. Corol. 2. and Prop. 3. Corol. 1.) Confequently the Curve $AHIK$, &c. which the Body moves in. whether thrown horizontally or obliquely, is a Parabola. Q. E. D.

Lemma 1. The Quotient which arifes from the Divifion of the Square of the Line AC by the Line AG, *viz.* the Quantity $\frac{ACq}{AG}$ (in either of the Parabolic Curves, *Fig.* 14. or 15.) or the Square of the Line AD divided by AM, *viz.* $\frac{ADq}{AM}$, or the Square of AE divided by AN, viz. $\frac{AEq}{AN}$ is equal to the *Parameter* of the Point A, for GHq is equal to AG multiplied by the *Parameter* (De L'*Hofpital* Con. Sect. B. 1. Prop. 1. & 3.) therefore the Parameter is equal to GHq divided by AG, that is, $\frac{ACq}{AG}$. The fame is demonftrable of AD divided by AM, &c. and confequently any of thefe Quanties may be indifferently put to exprefs the Parameter of the fame Point.

Lemma 2. The Velocity a Body would acquire, by falling from an Height equal to the fourth Part of the Parameter of the Point A, is to the Velocity it would acquire by falling from A to N, as AE is to twice AN.

Dem.

Dem. Since we are comparing the Velocity, which a Body would acquire by falling through a fourth Part of the Parameter, with that which it would acquire by falling to N, let $\frac{AEq}{AN}$ be made choice of to denote the Parameter. Then $\frac{\frac{1}{4}AEq}{AN}$ will express a fourth Part of the Parameter. Now because the Velocities, acquir'd by falling Bodies, are as the square Roots of the Spaces they fall through (Chap. V. §. 6.), the Velocity, acquir'd by a Body in descending through $\frac{\frac{1}{4}AEq}{AN}$ is to that Velocity, which it would acquire by falling through AN, as the square Root of $\frac{\frac{1}{4}AEq}{AN}$ to the square Root of AN; that is, extracting the Roots of those Quantities, as $\frac{\frac{1}{2}AE}{\sqrt{AN}}$ to $\sqrt{AN}$,

and, multiplying each Term by $\sqrt{AN}$. as $\frac{1}{2}AE$ to AN, or as AE to twice AN. Q. E. D.

Prop. The Velocity a Body ought to be projected with, to make it describe a given Parabola, is such as it would acquire by falling through a Space equal to the fourth Part of the Parameter belonging to that Point of the Parabola, from whence it is intended to be projected.

Dem. The Velocity with which a Body must be projected from A towards B, to make it describe the given Parabola $AHIK$, must be such, as would carry it to C by an uniform Motion, in the same Time that it would descend by its Gravity from A to G; and to E in the Time it would fall to N, &c. as was before observed. Now the Velocity, with which the Line AE is described with an uniform Motion, is to that which is acquired by the Body in falling to N in the same Time, as AE is to twice AN; because (Chap. V. §. 7.) its Velocity in N would have carried it over twice AN in that Time, had it also been uniform. But by *Lemma* 2. the Velocity a Body would acquire, by falling through a Space equal to a fourth Part of the Parameter of the Point A, is to that which it would acquire by falling from A to N, also as AE to twice AN. Since therefore the Velocity, with which the Line AE is described (or, which is the same Thing, that whereby the Body is projected) and that which a Body would acquire by falling through a fourth Part of the Parameter of the Point A,

A, bear one and the same Proportion to that Velocity which a Body would acquire by falling from *A* to *N*, they must be equal. 2. E. D.

Corol. This affords us an easy Method of finding what Direction it is necessary to throw a Ball in with a given Velocity, in order to strike an Object in a given Situation. *v. g.* Let it be requir'd to strike an Object as *K*, with a Ball thrown from *A* with a given Velocity. Here it is only necessary to make the Triangle *ANK* (suppose a right Line drawn from *A* to *K*) such that $\frac{NKq}{AN}$ or which is the same Thing $\frac{AEq}{EK}$ in the Triangle *AEK*, may be equal to four Times the Space a Body must fall through, to acquire such a Degree of Velocity as that with which it is intended to be thrown, and then *AE* will be the Direction sought. In order to this we must lay down the following Lemma.

Let there be a Circle as *ABC* (*Fig.* 16.) *AK* a Tangent in the Point *A*, *AB* and *KI* parallel to each other, and let the other Lines be drawn, as in the Figure, I say $\frac{AEq}{EK} = AB$.

For the Angle *ABE* is equal to the Angle *EAK* (32. *Elem.* 3.) and the Angle *BAE* is equal to the Angle *AEK* as alternate, therefore the Triangles *ABE* and *AEK* are similar; consequently *AB* is to *AE*, as *AE* to *EK*, and multiplying the extreme Terms together, and middle Terms together, *AB* × *EK* = *AEq* and dividing both Sides of the Equation by *EK*, $AB = \frac{AEq}{EK}$. Q. E. D. By the same Method of arguing $\frac{AIq}{IK}$ may be proved equal to *AB*.

The PROBLEM.

Let it be requir'd to strike a given Object as *K* (*Fig.* 17.) with a Ball projected from *A* with a given Velocity.

Solution. Erect *A B* perpendicular to the Horizon, and equal to four Times the Height a Body must fall from, to acquire the Velocity with which the Ball is to be thrown; bisect this in the Point *G*, through which draw *HC* perpendicular to *AB*, and meeting the Line *AC* (perpendicular to *AK*) in *C*. On *C* as a Center with the Radius *CA*, describe the Circle *ABD*; lastly, through *K* draw the Line *KEI* perpendicular

II. THE greatest horizontal Diftance, to which a Body can be thrown with a given Velocity, is at the Elevation of 45 Degrees *.

III. IF two Balls are thrown at different Elevations (but with equal Degrees of Velocity) the one as much above forty five Degrees as the other below, the horizontal Diftances (or Randoms) where they both fall, will be the fame †.

IV. THE

pendicular to the Horizon, cutting the Circle in the Points E and I; I fay AE or AI will be the Direction fought.

For by the Lemma, $AB = \frac{AEq}{EK}$ or $\frac{AIq}{IK}$, but (*ex conftructione*) AB is equal to four Times the Height a Body muft fall from, to acquire the Velocity with which it is to be thrown, therefore its Equal $\frac{AEq}{EK}$ or $\frac{AIq}{IK}$, is the fame, which by the Corollary was the Thing requir'd to determine the Direction fought; confequently the Parabola, which the Body will defcribe, will pafs through the Point K. Q. E. D.

Coroll. 1. From hence it is evident, that if the Object to be ftruck be placed any where in the horizontal Line AO (*Fig.* 18.) beyond Q the Problem is impoffible; for then QH will not touch the Circle, and the Ball will not reach that Point with any Direction whatever.

● And that when the Ball is directed towards H, it will fall on Q the greateft Diftance it can poffibly be thrown to; but the Angle QAH being equal to ABH in the oppofite Segment (32. *Elem.* 3.) is equal to half AGH at the Center (20. *Elem.* 3.) which is a right one; confequently QAH is an Angle of 45 Degrees.

† *Coroll.* 2. If the Object is fituated in the horizontal Line AO (*Fig.* 19.) but nearer to A, than the greateft horizontal Diftance at which it may be ftruck, fuppofe in K; the two Directions AE and AI, with which it may be hit, are equally diftant

IV. The Height a Body will rise to, when thrown perpendicularly upwards, is equal to half the greateſt horizontal Diſtance it can be thrown to, with the ſame Velocity *.

From hence we may eaſily know how far a Mortar-Piece, or other ſuch Machine, will carry a Ball. Let the Ball be ſhot perpendicularly upwards, note the Time of its Aſcent and Deſcent, half that is the Time of Deſcent, from whence we learn the Height from which it falls (for Bodies are obſerv'd to fall in the firſt Second of Time ſixteen Feet, conſequently in two Seconds they fall four Times ſixteen Feet (Chap. V. §. 4.) in three, nine Times as much, &c.) but the perpendicular Height from whence it falls is the ſame with that to which it aſcended, conſequently (§. 4.) the double of this is equal to the greateſt horizontal Diſtance to which that Machine will carry the Ball with an equal Charge.

diſtant from the Direction *AH*; for the Angles *IAH* and *HAE*, are equal, as inſiſting on equal Arches *IH* and *HE*, (28. *Elem.* 3.)

* *Coroll.* 3. The Altitude of a perpendicular Projection is equal to a fourth Part of the Height *AB*; for the Velocity, with which the Body is projected, is *(ex hypoth.)* ſuch as it would acquire by falling through a fourth Part of the Line *AB*; but a fourth Part of the Line *AB* is equal to half the Line *GH*, or *AQ* *(Fig.* 18.*)* that is, half the greateſt horizontal Diſtance to which the Body can be thrown.

See *Cotes*'s Harmonia Menſurarum, p. 87. *Keil*'s Introduct. ad Phyſ. Præl. 16.

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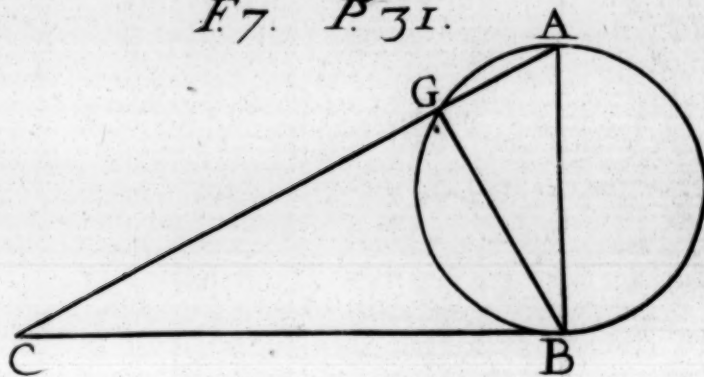
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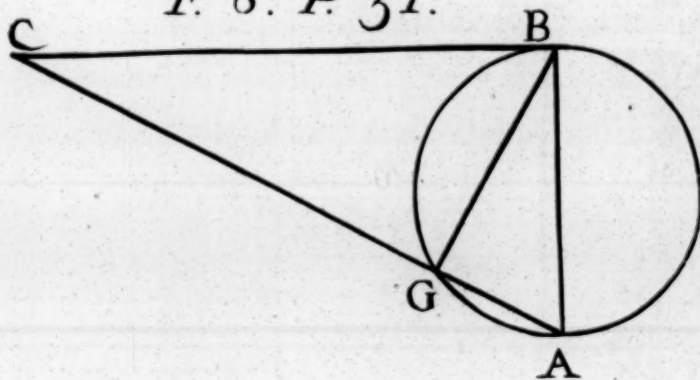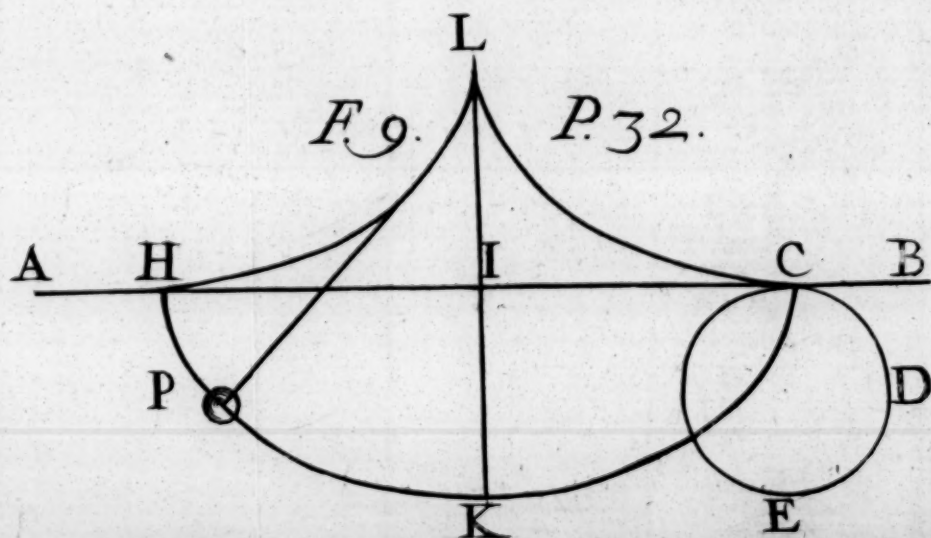
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V. The Randoms of two Projectiles, having the same Degrees of Elevation, but thrown with different Velocities, are as the Squares of the Velocities: For by the last, the Randoms are equal to double the Heights to which the Bodies thrown perpendicularly upwards will ascend, but the Heights are (Chap. V. §. 6.) as the Squares of the Velocities, therefore the Randoms are so too.

VI. Supposing the Motion of the Earth, all Bodies, when thrown perpendicularly upwards, describe *Parabola's*; notwithstanding they appear both to ascend and descend in the same right Line.

This may very easily be illustrated in the following Manner; let there be a Body carried uniformly along the Line AB *(Fig.* 20.*)* by the Motion of the Earth from A towards B; as it passes the Point C let it be projected upwards, by some Force acting underneath it in the Direction CO perpendicular to the former; the Body will not thereby lose its Motion, which it had in common with the Earth, towards B (by the first Law of Nature) but will be carried by two Motions, one towards B, the other towards O; let us then suppose, that in the Time it would have advanced forwards to P in the Line AB, it rises upwards to M in the Line CO; it will then be found in D (Chap. IV. §. 2.) In like Manner, supposing it would have advanced forward to Q

F while

while it rifes to N, it would then be found in E, afterwards in F, then in G, &c. defcribing the Curve CGL which (from what was demonftrated under §. 1.) is a Parabola *.

THE Reafon, why it appears to a Spectator to rife and fall perpendicularly, is becaufe he is carried uniformly along with it by the Motion of the Earth in the Direction AB. *v. g.* Suppofe the Spectator at C at the Inftant the Body is thrown from thence, when it arrives at D, he will be moved to P, when the Body is at E he will be at Q, &c. as is evident from what was obferved about the Motion of the Body in the Curve ; and they will both meet in L. Therefore fince the Spectator imagines himfelf ftanding ftill, and fees the Body always perpendicularly over his Head, he muft of Courfe think, that it rifes right up, and falls right down.

IT may be proper to obferve here, that Experiments, relating to the Motion of projected Bodies, do not exactly anfwer the Theory, the Refiftance of the Air deftroying Part of their Motion ; for which a fmall Allowance is to be made.

* *Dem.* Suppofe the Motion the Body had in common with the Earth towards *B (Fig.* 21.*)* and that with which it is projected towards *O*, fuch, as being compounded (Ch. IV. §. 2) would have produced a Motion in the Direction *CX*; it will follow from thence, that the Path defcribed by it will be the fame, as if it had been thrown in that Direction from a Point as *C* at reft; but in that Cafe it would have defcrib'd a Parabola as *CGL* (§. 1.) therefore alfo in this. *2. E. D.*

C H A P.

C H A P. VIII.

Of Centripetal and Centrifugal Forces.

WHEN a Body is projected in an horizontal Direction, and by its Gravity made to defcribe a Parabola, as demonftrated Chapter the laft; the Curvature of that Parabola will vary in Proportion to the Velocity with which the Body is thrown, and the Gravity which impels it towards the Earth. For the lefs its Gravity is in Proportion to the Quantity of Matter it contains, or the greater the Velocity is with which it is projected; the lefs it will deviate from a ftraight Line, and the further it will go, before it falls to the Earth. For Inftance, if a Bullet be fhot out of a Cannon from the Top of a Mountain with a given Velocity in an horizontal Direction, and goes in a Curve Line, fuppofe to the Diftance of two Miles from the Foot of the Mountain before it falls to the Ground; the fame Bullet, fhot with a much greater Velocity, would fly to a much greater Diftance before its Fall. And by encreafing the Velocity, the Diftance to which it is projected may be encreafed as much as you pleafe; fo that it will not fall to the Ground, till it is arrived at the Diftance of ten, or thirty, or ninety

F 2 Degrees;

Degrees; or till it has even furrounded the whole Earth, and arrives at the very Top of the Mountain from whence it was projected: In this Cafe it will perform a fecond Revolution, and fo on *in infinitum* without a new Projection, provided the Refiftance of the Air is taken away. Nay it may be projected with fuch Violence, that it will continually recede from the Earth, moving in a Curve, till at length it gets out of the Sphere of the Earth's Attraction; after which it will go on in a ftraight Line without ever returning. Which may thus be illuftrated.

LET ABC *(Fig. 22.)* reprefent the Earth, M the Top of the Mountain from whence the Body is projected in the Direction MQ: It may be thrown with fuch Force as to carry it to B before it falls, or to C, or even to go round to M, defcribing the Circle MDM; or laftly, it may be made to defcribe the Curve MO, till it gets out of the Sphere of the Earth's Attraction, fuppofe at O, going on afterwards in the infinite ftraight Line OX, there being nothing to ftop or alter its Courfe. Farther it may be projected with fuch a Force from M *(Fig. 23.)* as will caufe it continually to recede from the Earth, till it arrives at the oppofite Point G, defcribing the Curve MKG; and if the Point G is within the Sphere of the Earth's Attraction, the Body will return to M, defcribing the Curve GLM exactly fi-
milar

milar to MKG ; and in moving nearer and nearer to the Earth, till it comes to M, will regain what Velocity it loft in going from M to G, its Gravity confpiring with its Motion from G to M in the fame Degree in which it oppofed it from M to G ; confequently the Body when at M, having recovered the Velocity with which it fet out, will be inabled to perform a fecond Revolution in the fame Curve as before; and fo on.

AGAIN, fuppofe it had been projected from the Point M, with a lefs Degree of Force than would have carried it round in the Circle MDM *(Fig.* 22.*)* but greater than would have fuffered it to have fallen to the Earth at the oppofite Point F (*Fig.* 23.) it would alfo in this Cafe have arrived at the Point M from whence it fet out; for the Excefs of Velocity it would have gained in F, by its Tendency towards the Earth in its Way thither, over and above that, with which it was projected from M, would be fufficient to carry it off again from the Earth, till it arrived at M ; and to make it defcribe the Path FPM exactly fimilar and equal to the former, lofing in its Way from F to M juft fo much Velocity, as it gained by paffing from M to F ; and thereby it would be inabled to perform an infinite Number of Revolutions in the fame Curve, without requiring a fecond Projection.

FROM

FROM hence it follows, that suppofing a Body projected from a Point at any Diftance within the Sphere of the Earth's Attraction, with a Force fufficient to carry it half round without falling to the Surface, it is impoffible it fhould fall upon any Part of the other half; but will return to the Point from whence it fet out, making continual fucceffive Revolutions in the fame Curve; provided it meets with no Refiftance from the Medium through which it paffes, nor any other Obftacle to obftruct its Motion *.

FROM hence alfo it is clear, that, the nearer the revolving Body approaches to the Earth, the fafter it moves; its Velocity being continually increafed during the Time of its Accefs towards the Earth, and as much retarded during its Recefs from it. And this Acceleration and Retardation will always be fuch, that the Body will defcribe equal Areas in equal Times: The Meaning of which is, that if we imagine a Line conftantly extended from the Center of the Earth to the Center of the Body, that Line will always defcribe or pafs through equal Surfaces or Spaces in equal

* Gravity is here fuppofed to be inverfly as the Squares of the Diftances from the Earth, for 'tis poffible that the Force, by which a Body tends towards another, may vary in fuch a Manner at different Diftances, that the projected Body fhall defcribe a fpiral Line, continually approaching to, or receding from that about which it revolves.

Times,

Times, for it conftantly becomes fhorter the fafter it moves, and *vice versâ* *.

AND for the fame Reafon that a Body, projected with a fufficient Velocity, may by the Force of Gravity be made to defcribe a Curve round the Earth, and perform continual fucceffive Revolutions therein ; it follows, that the Moon may by the fame Force of Gravi-

* *Dem.* Let the Time in which the Body performs one Revolution be divided into equal Parts, in the firft of which let the Body defcribe the right Line *AB* (*Fig.* 24.) in the fecond Part of Time, if not prevented, it would go ftraight on to *c,* defcribing the Line *B c* equal to *AB* by the firft Law of Nature; the Lines *SA, SB, Sc* being drawn, the Triangles *SBA, ScB,* will be equal to each other, their Bafes *AB* and *Bc* being equal, and their Heights *S* the fame (38. *Elem.* 1.) When the Body arrives at *B,* let the centripetal Force by one fingle Impulfe turn it out of the ftraight Line *B c* into the Line *BC;* in which let it move on uniformly without receiving a fecond Impulfe till it comes to *C.* Let *Cc* be drawn parallel to *SB* meeting *BC* in *C;* then at the End of the fecond Part of Time, the Body will be found in *C,* having defcribed the Diagonal of the Parallelogram *Nc* (Chap. IV. §. 2.) Draw *SC,* and the Triangle *SCB* will be equal to the Triangle *ScB,* (each having the fame Bafe *SB* and being between the fame Parallels *Cc* and *SB*) and therefore alfo equal to the Triangle *SBA.* For the fame Reafon, if the centripetal Force acts in the Points *C, D, E* fucceffively, fo as to make the Body defcribe the ftraight Lines *CD, DE, EF,* &c. in fo many equal Parts of Time, the Triangles *SCD, SDE, SEF,* &c. will be all equal to one another and to the Triangle *SAB.* Confequently equal Areas are defcribed in equal Times. Let us then fuppofe the Bafes of thofe Triangles, *viz. AB, BC, CD, DE,* &c. diminifhed *in infinitum,* and likewife the Times in which they are defcribed; then will the Perimeter *A, B, C, D, E, F,* &c. become a Curve, and any Number of thofe Triangles taken together (or their Areas) will be proportionable to the Times in which they are defcribed. *Q. E. D.*

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ty be made to revolve about the Earth, or any other Planet by the like Force, about the Sun; if the Velocities with which they move are duly adjusted to the Forces by which they are acted upon.

WHEN a Body revolves about another in this Manner, that Force or Power by which it is prevented from flying off (as it otherwise would do in a Tangent to the Curve which it describes) is call'd the *Centripetal*; the Counter-action of this, by which it endeavours to fly off, the *Centrifugal*; these, by the third Law of Nature being equal to each other, are called by one common Name, *Central Forces*; that with which the Body is at first projected, or continues its Motion from any Point, is the *Projectile* Force; and the Time in which it performs one Revolution, the *Periodical* Time.

THESE Forces, properly relating to the Motions of the Heavenly Bodies, will be more largely treated of in another Place.

CHAP. IX.

Of the Communication of Motion.

I. BEFORE we proceed to explain the Laws, by which Bodies communicate their Motion from one to another, it is very

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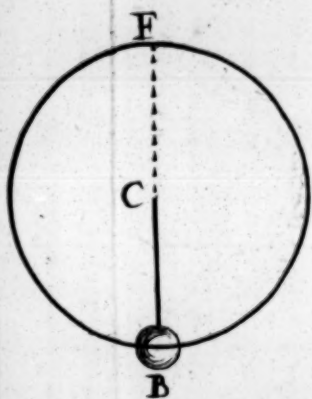
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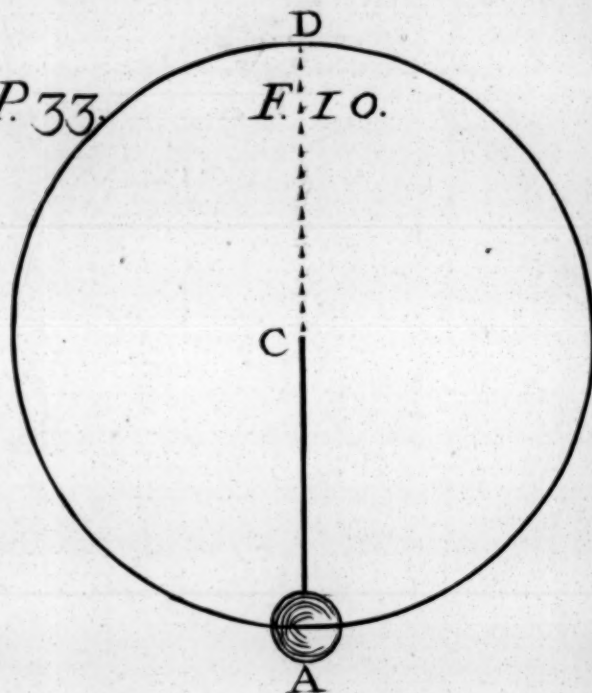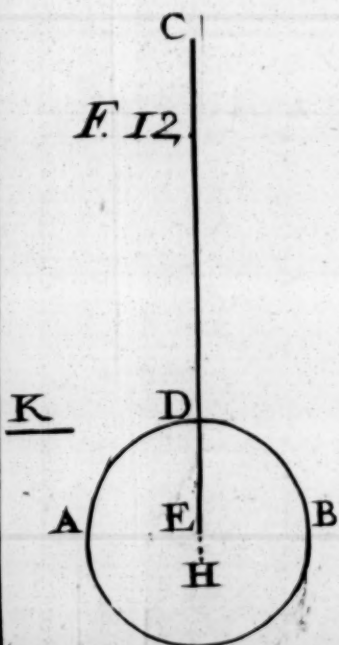
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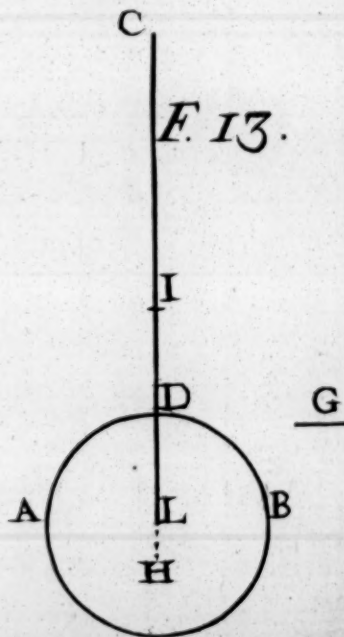
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neceffary to make a Diftinction between Motion and Velocity; which ought to be well obferv'd, and is as follows.

By the Motion of a Body (fometimes called its Quantity of Motion, fometimes its *Momentum*) is not to be underftood the Velocity only, with which the Body moves; but the Sum of the Motion of all its Parts taken together: Confequently the more Matter any Body contains, the greater will be its Motion, though its Velocity remains the fame. Thus, fuppofing two Bodies, one containing ten Times the Quantity of Matter the other does, moving with equal Velocity; the greater Body is faid to have ten Times the Motion, or Momentum, that the other has: For 'tis evident, that a tenth Part of the larger has as much, as the other whole Body. In fhort, that Quality in moving Bodies, which Philofophers underftand by the Term Momentum or Motion, is no other than what is vulgarly called their *Force*, which every one knows to depend on their Quantity of Matter, as well as their Velocity. This is that Power, a moving Body has to affect another in all Actions that arife from its Motion, and is therefore a fundamental Principle in Mechanics.

II. Now, fince this Momentum, or Force, depends equally on the Quantity of Matter a Body contains, and on the Velocity with which it moves; the Method, to determine how

great

great it is, is to multiply one by the other. Thus, fuppofe two Bodies, the firft having twice the Quantity of Matter, and thrice the Velocity which the other has; any two Numbers, that are to each other as two to one, will exprefs their Quantities of Matter (it being only their relative Velocities and Quantities of Matter which we need confider) and any two Numbers that are as three to one their Velocities; now multiplying the Quantity of Matter in the firft, *viz.* two by its Velocity three, the Product is fix; and multiplying the Quantity of Matter in the fecond by its Velocity, *viz.* one by one, the Product is one; their relative Forces therefore or Powers will be as fix to one, or the Moment of one is fix Times greater than that of the other. Again if their Quantities of Matter had been as three to eight and their Velocities as two to three, then would their Moments have been as fix to twenty four, that is, as one to four.

Th is being rightly apprehended, what follows, concerning the Laws of the Communication of Motion by Impulfe, and the mechanical Powers, will be eafily underftood.

The Communication of Motion.

I. In Bodies not Elaftic.

III. Those Bodies are faid to be not *Elaftic*, which, when they ftrike againft one another,

ther,

ther, do not rebound, but accompany one a-
nother after Impact, as if they were joined.
This proceeds from their retaining the Impref-
fion, made upon their Surfaces, after the im-
preffing Force ceafes to act. For all rebound-
ing is occafioned by a certain Spring in the
Surfaces of Bodies, whereby thofe Parts, which
receive the Impreffion made by the Stroke,
immediately fpring back, and throw off the
impinging Body; now, this being wanting in
Bodies void of Elafticity, there follows no Se-
paration after Impact.

IV. When one Body impinges on another
which is at reft, or moving with lefs Velocity
the fame Way, the Quantity of the Motion
or Momentum in both Bodies taken together
remains the fame after Impact, as before; for
by the third Law of Nature, the Reaction of
one being equal to the Action of the other,
what one gains, the other muft lofe.

Thus, fuppofe two equal Bodies, one im-
pinging with twelve Degrees of Velocity on
the other at reft : The Quantities of Matter in
the Bodies being equal, their Moments and
Velocities are the fame; the Sum in both
twelve; this remains the fame after Impact
(§. 4.) and is equally divided between them
(§. 3.) they have therefore fix a-piece, that is,
the impinging Body communicates half its
Velocity, and keeps half.

G 2 V. When

V. When two Bodies impinge on each o-
ther by moving contrary Ways, the Quantity
of Motion, they retain after Impact, is equal
to the Difference of the Motion they had be-
fore; for by the third Law of Nature, that,
which had the least Motion, will destroy an
equal Quantity in the other, after which they
will move together with the Remainder, that
is the Difference.

Thus for Instance, let there be two equal
Bodies moving towards each other, the one
with three Degrees of Velocity, the other with
five, the Difference of their Moments or Velo-
cities will be two; this remains the same after
Impact (§. 5.) and is equally divided be-
tween them (§. 3.) they have therefore one
a-piece: That is, the Body, which had five
Degrees of Velocity, loses three or as much as
the other had, communicates half the Remain-
der, and keeps the other half *.

* From these Positions it is easy to reduce a Theorem, that
shall shew the Velocity of Bodies after Impact in all Cases what-
ever. Let there be two Bodies A and B, the Velocity of the first
a, of the other b; then (§. 2.) the Moment of A will be ex-
pressed by Aa, and of B by Bb; therefore the Sum of both
will be $Aa + Bb$; and $Aa - Bb$ will be the Difference when
they meet. Now these Quantities (by §. 4. and 5.) remain the
same after Impact; but knowing the Quantities of Motion and
Quantities of Matter, we have the Velocity (which §. 3. is the
same in both) by dividing the former by the latter (as follows
from §. 2.) therefore $\dfrac{Aa + Bb}{A + B}$ or $\dfrac{Aa - Bb}{A + B}$ will in all Cases ex-
press the Velocity of the Bodies after Impact.

II. In

II. In Elastic Bodies.

VI. BODIES perfectly *Elastic* are such as rebound after Impact with a Force equal to that with which they impinge upon one another: Those Parts of their Surfaces, that receive the Impression, immediately springing back, and throwing off the impinging Bodies with a Force equal to that of Impact.

VII. FROM hence it follows, that the Action of Elastic Bodies on each other (that of the Spring being equal to that of the Stroke) is twice as much as the same in Bodies void of Elasticity. Therefore, when Elastic Bodies impinge on each other, the one loses, and the other gains twice as much Motion as if they had not been Elastic; we have therefore an easy Way of determining the Change of Motion in Elastic Bodies, knowing first what it would have been in the same Circumstances, had the Bodies been void of Elasticity.

THUS, if there be two equal and Elastic Bodies, the one in Motion with twelve Degrees of Velocity impinging on the other at rest, the impinging Body will communicate twice as much Velocity as if it had not been Elastic, that is (by §. 4.) twelve Degrees, or all it had; consequently it will be at rest, and the other will move on with the whole Velocity of the former.

VIII. IT

VIII. It sometimes happens, that in Bodies not Elastic, the one loses more than half its Velocity, in which Case, supposing them Elastic, it loses more than all; that is, the Excess of what it loses, above what it has, is negative, or in a contrary Direction; thus, suppose the Circumstances of Impact such, that a Body, which has but twelve Degrees of Velocity, loses sixteen; the overplus four is to be taken the contrary Way, that is, the Body will rebound with four Degrees of Velocity. *v. g.* Let it be required to determine the Velocity of a Body after Impact against an immoveable Object. Let us first suppose the Object and Body both void of Elasticity : 'Tis evident the impinging Body would be stopp'd or lose all its Motion, and communicate none; if they are Elastic, it must lose twice as much (by §. 7.) and consequently will rebound with a Force equal to that of the Stroke.

IX. It is sufficient if only one of the Bodies is Elastic, provided the other be infinitely hard; for then the Impression in the Elastic Body will be double of what it would have been, had they both been equally Elastic : And consequently the Force, with which they rebound, will be the same as if the Impression had been equally divided between the two Bodies.

X. There are no Bodies, that we know of, either perfectly Elastic, or infinitely hard;

the

the nearer therefore any Bodies approach to Perfection of Elasticity, so much the nearer do the Laws, which they observe in the mutual Communication of their Motion, approach to those we have laid down.

XI. Sir Isaac Newton made Trials with several Bodies, and found that the same Degree of Elasticity always appeared in the same Bodies, with whatever Force they were struck; so that the Elastic Power, in all the Bodies he made Trial upon, exerted itself in one constant Proportion to the compressing Force. He found the Celerity with which Balls of Wool, bound up very compact, receded from each other, to bear nearly the Proportion of five to nine to the Celerity wherewith they met; and in Steel, he found nearly the same Proportion; in Cork the Elasticity was something less, but in Glass much greater; for the Celerity, with which Balls of that Material separated after Percussion, he found to bear the Proportion of fifteen to sixteen to the Celerity wherewith they met *.

XII. We have hitherto supposed the Direction, in which Bodies impinge upon one another, to be perpendicular to their Surfaces: When it is not so, the Force of Impact will be less, by how much the more that Direction varies from the Perpendicular; for it is manifest that a direct Impulse is the greatest of all

* Newt. Princip. Phil. pag. 21.

others

others that can be given with the same Degree of Velocity *.

XIII. This is the Case, when Bodies impel one another by acting upon their Surfaces; but in Forces, where the Surfaces of Bodies are not concerned, as in Attraction, &c. we must not consider the Relation which the Direction of the Force has to the Surface of the Body to be moved, but to the Direction in which it is to be moved by that Force. Here the Force of Action will be less, by how much the more these two Directions vary from each other †. My Meaning in both Cases

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* The Force of oblique Percussion is to that of direct, as the Sine of the Angle of Incidence to the Radius.

Dem. Let there be a Plane as *AD* (*Fig.* 25.) against which let a Body impinge in the Point *D* in the Direction *BD :* which Line may be supposed to express the Force of direct Impulse, and may be resolved into two others (Chap. IV. §. 2) *BC* and *BA*; the one parallel, the other perpendicular to the Plane; but that Force which is exerted in a Direction parallel to the Plane can no Way affect it; the Stroke therefore arises wholly from the other Force expressed by the Line *BA*; but this is to the Line *BD*, as the Sine of the Angle of Incidence *ADB* to the Radius; from whence the Proposition is clear.

If the Surface of the Body to be struck is a Curve, then let *AD* be made a Tangent to *D* the Point of Incidence, and the Demonstration will be the same.

† The Force of oblique Action is to that of direct, as the Co-Sine of the Angle comprehended between the Direction of the Force, and that wherein a Body is to be moved thereby, to the Radius.

Dem. Let *FD* (*Fig.* 26.) represent a Force acting upon a Body as *D*, and impelling it towards *E*; but let *DM* be the only Way in which it is possible for the Body to move; the
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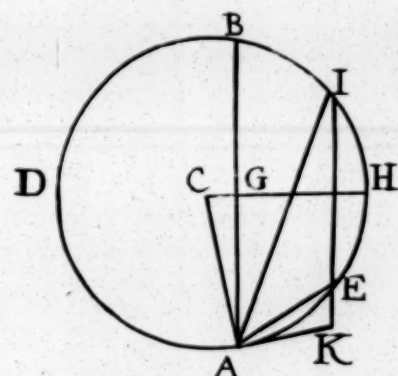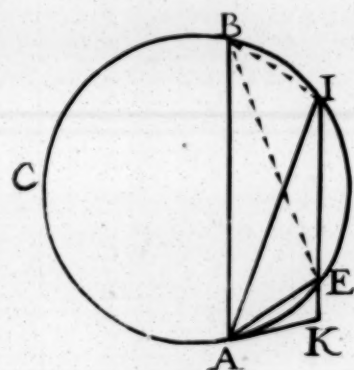
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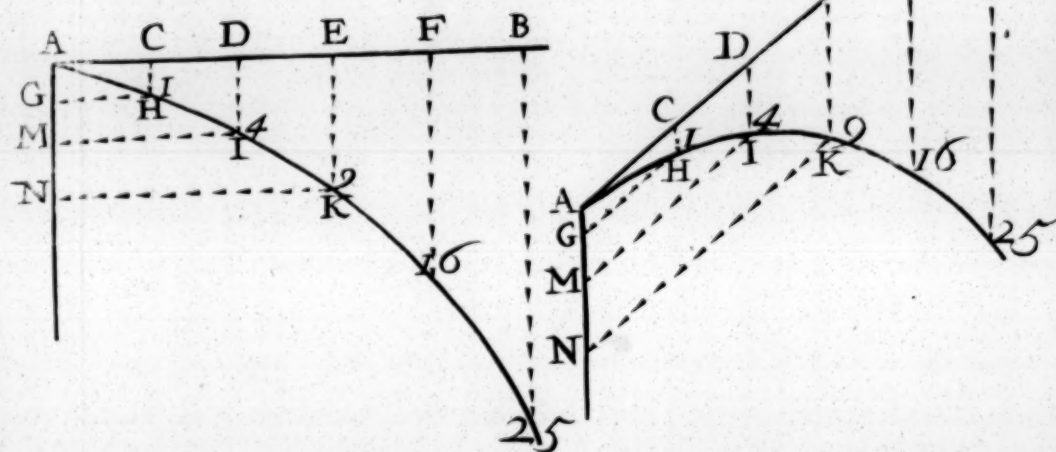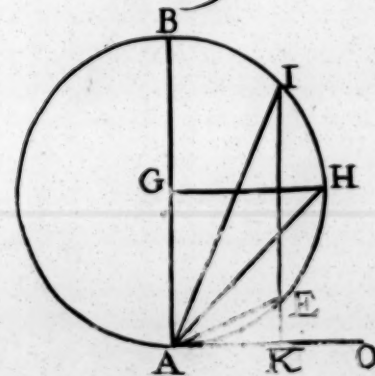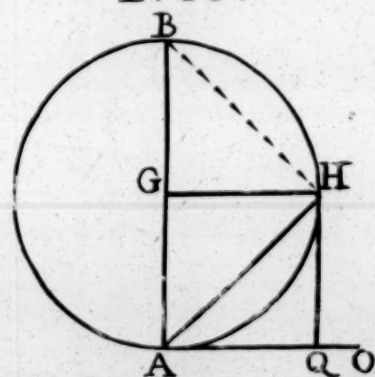
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Cafes will be underftood from the Inftance of a Ship under Sail. The Force, by which the Wind acts upon the Sail, will be lefs, by how much the more its Direction varies from one that is perpendicular to its Surface: But the Force of the Sail, to move the Ship forward, will be lefs, by how much the more the Direction of the Ship's Courfe varies from that, in which fhe is impell'd by the Sail.

XIV. To this we may add the following Propofition, relating to oblique Forces, *viz.* that, if a Body is drawn or impelled three different Ways at the fame Time by as many Forces acting in different Directions; and if the Quantity of thofe Forces is fuch, that the Body is kept in its Place by them: Then will the Forces be to each other, as the feveral Sides of a Triangle, drawn refpectively parallel to the Directions in which they act *.

Force *FD* may be refolved (Chap. 4. §. 2.) into two others *FG* and *FH*, or which is equal to it *GD*; but 'tis evident that only the Force *GD* impels it towards *M*. Now, *FD* being the Radius, *GD* is the Co-Sine of the Angle *FDG* comprehended between the two Directions *FE* and *GM*; from whence the Propofition is clear.

* *Dem.* Let the Lines *AB*, *AD*, *AE*, (*Fig.* 27.) reprefent the three Forces acting upon the Body *A* in thofe Directions and by that Means keeping it at reft in the Point *A*. Then the Forces *EA* and *DA* will be equivalent to *BA*, otherwife the Body would be put into Motion by them (*contra Hypoth.*) But thefe Forces are alfo equivalent to *AC* (Chap. IV. §. 2) confequently *AC* may be made Ufe of to exprefs the Force *AB*; and *EC*, which is parallel and equal to *AD*, may exprefs the Force *AD*, while *AE* exprefes its own: But *ACE* is a Tri-

H   angle

CHAP. X.

Of the Mechanical Powers.

I. HAVING, in the foregoing Chapter, accounted for the Communication of Motion by Impulfe; we proceed next to confider Motion as communicated by Means of certain Inftruments, commonly known by the Names of *Mechanical Powers.* The Ufe of thefe Powers confifts chiefly in managing great Weights, or performing other Works with a determinate Force.

II. THEY are ufually reckoned five. *viz.* The Lever, the Wheel and Axis, the Pully, the Screw, and the Wedge; to which fome add the inclined Plane. To thefe all Machines how complicated foever are reducible.

III. THESE Inftruments have been of very ancient Ufe; for we find that *Archimedes* was well acquainted with the Extent of their Power; as may be inferred from that celebrated Saying of his, Δὸς πȣ̃ ϛῶ, καὶ τὴν γῆν κινήσω. By which he meant, that the greateft imaginable Weight might be moved with the fmalleft Power.

angle whofe Sides are all parallel to the given Directions; therefore the Sides of this Triangle will exprefs the Relation of the Forces by which the Body is kept at reft. *2. E. D.*

IV. THAT

IV. T H A T Body, which communicates Motion to another, is called the *Power*; that which receives it, the *Weight*.

V. T H A T Point in a Body, which remains at reft, while the Body is turning round, is called the *Center of Motion*. Befides this, there are two other Centers in Bodies, 1. that of *Magnitude*, which is a Point, as near as poffible, equally diftant from all the external Parts of the Body; 2. that of *Gravity*, or that, about which all the Parts of the Body, in whatever Situation it is placed, exactly balance each other.

VI. W H E N a Body communicates Motion to another, it lofes juft fo much of its own, as it communicates to that other; the Action of one being equal to the Reaction of the other. See Chapter the laft §. 4. and 5.

VII. W H E N two Bodies have fuch Relation to each other (fuppofe them fixed to different Parts of the fame Machine) that if one be put into Motion, the other will thereby neceffarily have fuch a Degree of Velocity given it, that their Moments will be equal; it will then be impoffible, that one fhould begin to move without communicating to the other a Quantity of Motion equal to its own; 'tis evident therefore from the laft Propofition, that if we fuppofe it to begin to move, in that very Inftant it muft lofe all its own Motion by communicating the whole to the other

Body,

Body, and therefore, being left to itfelf, will remain at reft, and communicate none at all.

Now the Moments of two Bodies are equal (Chap. IX. §. 2.) when the Velocity of the firft is to that of the fecond, as the Quantity of Matter of the fecond to that of the firft; for if we fuppofe their Quantities of Matter as one to three, then by the Suppofition their Velocities are as three to one; and if we multiply the Quantity of Matter in the firft, *viz.* one, by its Velocity three, and that of the other, *viz.* three, by its Velocity one; their Products are equal; their Moments are therefore by the Definition (Chap. IX. §. 1. and 2.) equal. They will alfo be equal, when the Spaces the Bodies pafs over are in that Proportion; for the Times they both move in being the fame, the Spaces will always be as the Velocities.

VIII. From hence it follows, that in any Machine, whether fimple or compound, the Power however fmall may have a Moment equal to that of the Weight; provided the Machine be fuch, that when it is in Motion, the Velocity of the Power fhall bear fuch Proportion to that of the Weight, as the Weight does to the Power; for then, what the Power wants in Quantity of Matter or Weight, will be made up in Velocity; confequently their Moments will be equal by §. the laft, and therefore by §. 7. they will exactly balance each other; or be in *Æquilibrio.*

IX. But

IX But if the Power bears a greater Proportion to the Weight, than the Velocity of the Weight to that of the Power ; it will then have a greater Momentum than the other, and confequently may communicate fuch a Momentum to it as it will receive, without lofing all its own ; the Remainder therefore, if fufficient to overcome the Friction of the Machine, will put it into Motion.

We proceed now to treat of each Mechanical Power in its Order, and

I. Of the Lever.

X. The Lever is a right Line (or Bar whofe Weight in Theory is not confider'd) moveable on a Center, which is called its *Fulcrum*, or *fixed Point*.

XI. The Æquilibrium in this Machine is, when the Diftance of the Power from the fixed Point is to that of the Weight from the fame, as the Quantity of Matter in the Weight is to that in the Power.

For, fuppofing the Lever placed on its Fulcrum with the Weight to be raifed at one End, and the Power applied to the other ; 'tis evident, the farther the Power is placed from the Fulcrum or Center of Motion, the larger will be its Sweep when the Machine is put in Motion ; that is, it will move over fo much more Space in the fame Time than the Weight to be raifed : now, if it is placed juft

fo

fo much farther from the Fulcrum, as it is
lefs than the Weight, it will move juft fo
much fafter; their Moments therefore will be
equal (§ 7.) and confequently the Power and
Weight will exactly balance each other, or be
in Æquilibrio *. And, if the Power is fuffi-
ciently augmented to overcome the Friction
of the Machine, it will put it in Motion.

THE Lever is of three Kinds. 1. When the
fixed Point is between the Weight and the
Power, as in the laft Cafe. 2. When the
Weight is between the fixed Point and the
Power. 3. When the Power is between the
fixed Point and the Weight.

IN all which Cafes the Æquilibrium will
be, when their Diftances from the fixed Point
are fuch, that their Velocities fhall be inverfe-
ly as their Quantities of Matter; for then by
(§ 7.) being at reft, neither of them will com-
municate any Motion to the other.

THE common Scales may be confidered as

* Geometrically thus. Let *AB* (*Fig.* 28.) reprefent the Le-
ver, *F* the Fulcrum, *W* the Weight, P the Power, the one fuf-
pended at the Extremity of the Lever *A*, the other at *B*, and
let *BF* be to *FA* as *W* to *P*; then while the Lever moves from
the Situation *AB* into that of *CD*, the Point *B* which fuftains the
Power will move as much farther than *A* which fuftains the
Weight (and confequently as much fafter fince they perform their
Motions in the fame Time) as the Arch *BD* is longer than *AC*;
that is, the Triangles *BFD* and *AFC* being fimilar, as the Arm
BF is longer than *AF*, which (*ex Hypoth.*) is as much as the
Weight exceeds the Power, they will therefore (§ 7.) be in Æ-
quilibrio. *Q. E. D.*

a Lever

a Lever of the first Kind, where the Weight and Power are applied at equal Distances from the fixed Point.

THE Steelyard is also a Lever of the first Kind, but whose Arms are unequal.

THE Difference between the Use of the Scales and the Steelyard consists in this; that as in one you make use of a larger Power (or more Weights) to estimate the Weight of an heavier Body ; in the other you use the same Power, but give it a greater Velocity with respect to that of the Weight by applying it farther from the fixed Point, which by § 7. will have the same Effect.

II. The WHEEL and AXIS.

XII. THIS Machine is a Wheel, that turns round together with its Axis; the Power in this is applied to the Circumference of the Wheel, and the Weight drawn up by means of a Rope wound about the Axis.

XIII. IN this there will be an Æquilibrium, when the Weight is to the Power, as the Diameter of the Wheel to the Diameter of of the Axis.

'TIS evident, the Velocity of the Power will exceed the Velocity of the Weight, as much as the Circumference of the Wheel exceeds that of its Axis ; because the Spaces they pass over in one Revolution will be as those Circumferences ; that is, as much as the Diameter

meter of one exceeds that of the other (the Circumferences of Circles being as their Diameters ;) what therefore in this Cafe the Power wants in Weight will be made up in Velocity, from whence (§ 7.) there will be an Æquilibrium *.

THE Ufe of this Machine is to raife Weights to greater Heights than the Lever can do, becaufe the Wheel is capable of being turned feveral Times round, which the Lever is not ; and alfo to communicate Motion from one Part of a Machine to another ; accordingly there are few compound Machines without it.

III. The PULLEY.

XIV. A Pulley is an Inftrument compofed of one or more Wheels moveable on their Axes.

XV. A fimple Pulley, if its Axis is fixed, is of no other Ufe, than to alter the Direction of the Power ; for the Power and Weight will both move through an equal

* Geometrically thus. Let *AB (Fig.* 29.*)* be the Diameter of the Wheel, *DE* that of the Axis, *W* the Weight, and *P* the Power ; when the Wheel begins to move, the Point *B* and *D* will defcribe fimilar Arches about the Center *C*, in the fame Manner the Point *A* and *B* in the Lever were fhewn to do about the fixed Point *F (Fig.* 28.*)* that is, the Point *B* will move as much fafter than *D*, as *CB* is longer than *CD* or *AB* than *DE*, the Motion therefore of *P* (§ 7.) will be equal to that of *W*. From whence the Propofition is clear.

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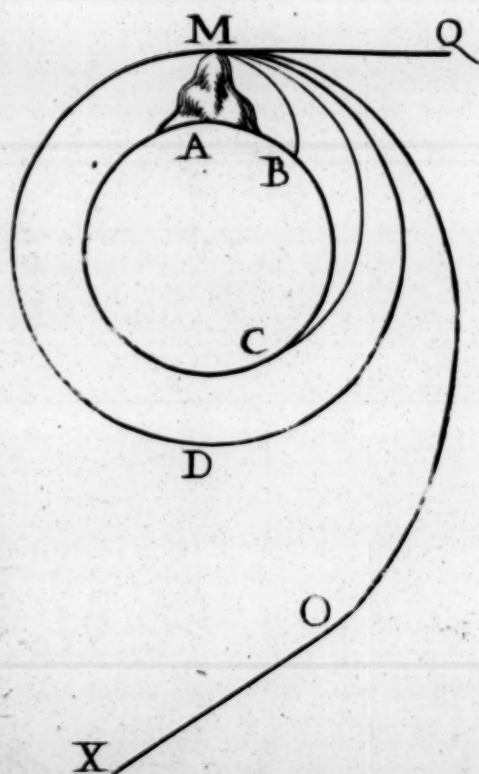, and
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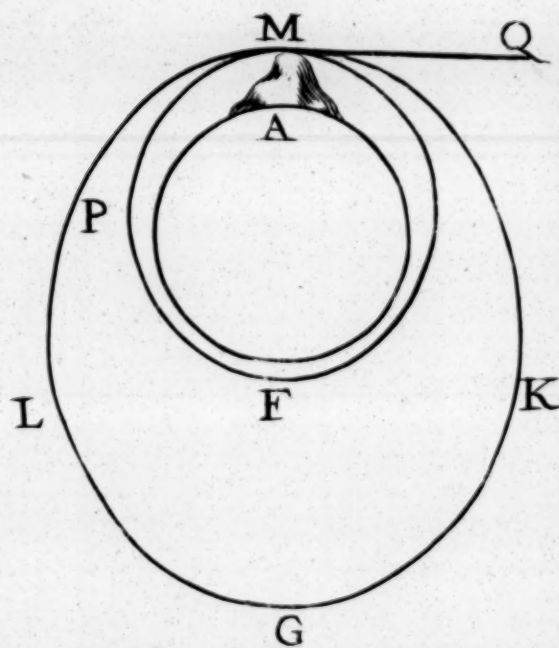
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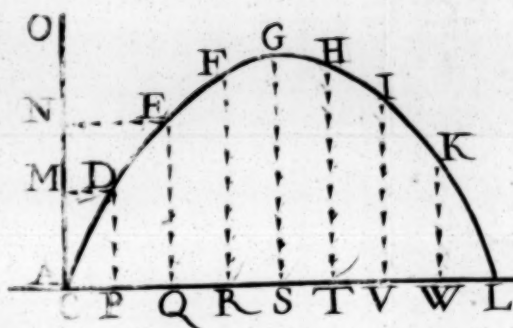
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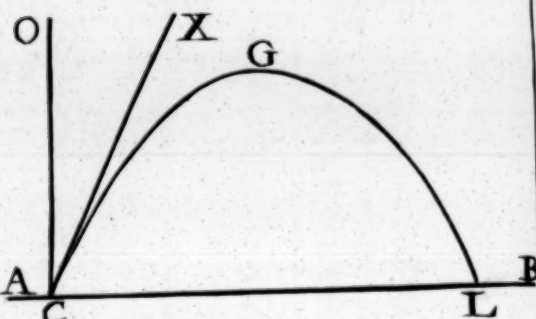
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Space in the fame Time. But in a Pulley not fixed, as in *Fig.* 30. where the Rope runs under it, or in a Combination of Pullies as in *Fig.* 31. the Æquilibrium will be, when the Power is to the Weight, as one to the Number of Ropes, that pafs between the upper and lower Pullies.

For, fuppofe one End of the Rope fixed in B (*Fig.* 30.) the other fupported by the Power P, it is evident, that in order to raife the Weight W one Foot, the Power muft rife two, for both Ropes, *viz.* BC and CP, will be fhortened a Foot apiece, whence the Space run over by the Power will be double to that of the Weight ; if therefore the Power is to the Weight as one to two, their Moments will be equal : for the fame Reafon, if there be four Ropes paffing from the upper to the lower Pullies as in *Fig.* 31. the Velocity of the Power will be quadruple to that of the Weight, or as four to one, *&c.* In all Cafes therefore when the Power is to the Weight, as one to the Number of Ropes paffing from the upper to the lower Pullies (§ 7.) there will be an Æquilibrium.

XVI. If the Pullies be difpofed as in Figure the 32d, each having its own particular Rope, the Action of the Power will be very much increafed ; for here every Pulley doubles it, wherefore the Power is four Times greater with two Pullies, eight Times with three,

I   fixteen

sixteen Times with four, &c. For, it is evi
dent from the Confideration of the Figure
the firft will move half as faft as the Power
the fecond half as faft as that, and fo on
wherefore (§. 7.) the Power is doubled by
each Pulley.

THE Ufe of the Pulley is nearly the fame
with that of the Wheel and Axis, but it i
more portable, and eafier to be fixed up.

IV. The SCREW.

XVII. IN this Machine the Æquilibrium
will be, when the Power is to the Weight
as the Diftance between any two contiguou
Threads or Spirals in the Screw, to the Way
defcribed by the Power in one whole Revolu
tion. It is manifeft from the Form of the
Machine *(Fig. 33.)* that in one Revolution o
the Screw, the Weight will be moved through
a Space equal to the Diftance of two conti
guous Threads, and that the Power will rui
through a Space equal to the Compafs it take
in one Revolution, therefore (§. 7.) if th
Weight exceeds the Power in this Proportion
there will be an Æquilibrium.

THIS Machine is of great Force, and very
ufeful in retaining Bodies in a compreffed
State, becaufe it will not run back, as the
three foregoing will, when the Power is re
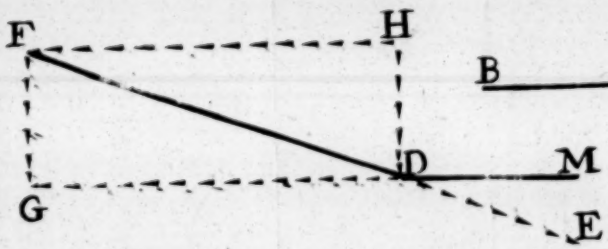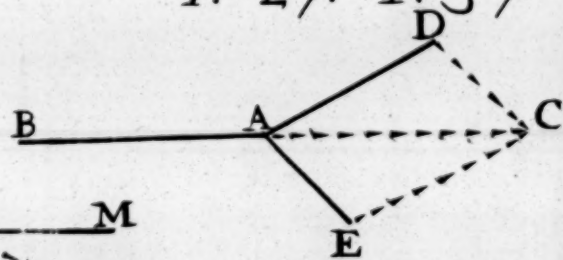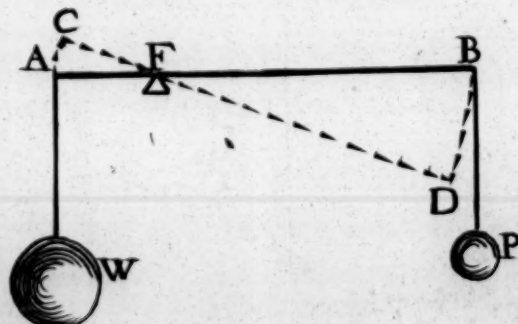moved. This arifes from the great Friction

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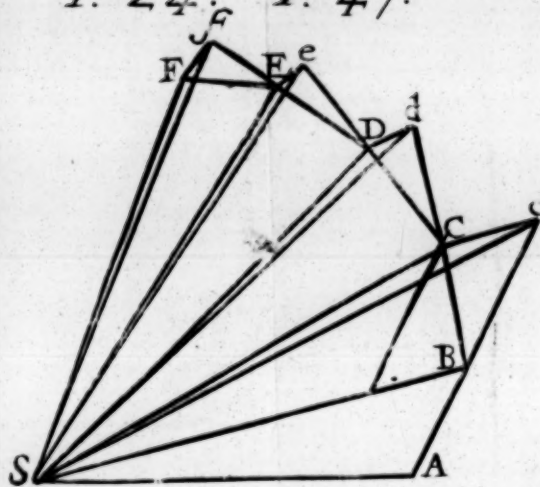
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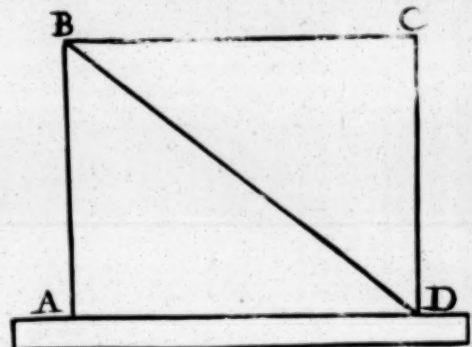
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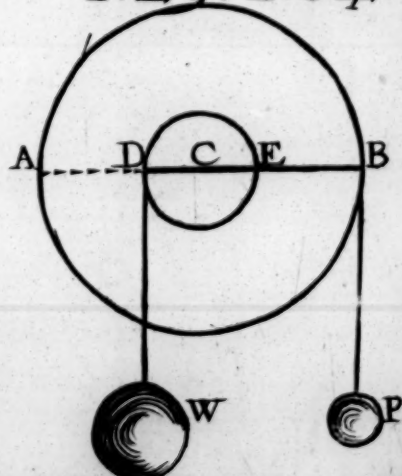
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of thofe Parts in the Screw, which during its Motion flide upon thofe that are at Reft.

V. The WEDGE.

XVIII. THIS Inftrument is formed by two equal Rectangles, joined at their lower Bafes, and feparated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*.

XIX. IN the foregoing Mechanical Powers we have all along confidered the Weight, as moved in the fame Direction with that, in which it is acted upon by the Machine, as is commonly the Cafe; but in this, the Weight is generally applied in fuch a Manner, as to be made to move in a Direction different from that, in which it is protruded by the Wedge; hence it is, that Mathematicians have widely differed in their Determination of the Power of this Machine, fome confidering the Weight as moved by it in one Direction, and fome in another. Nay, there are fome, even among the late Writers, that have been led into manifeft Errors by it. We will therefore lay down the feveral Proportions, they have given us, for the determining the Power of this Machine, and examine them one by one. 1. It is demonftrated by fome, that the Power will be equivalent to the Refiftance of the Weight, when it bears fuch Proportion to it, as the Breadth of the Back of the Wedge

does to the Sum of its Sides ; or, which is the fame Thing, as half that Breadth to one of its Sides.   2. Others make it fomewhat larger, and demonftrate, that it ought to be, as half the Breadth of the Back to the perpendicular Height of the Wedge.   3. Some are of Opinion, that there will not be an Æquilibrium in this Machine, unlefs the Power is to the Weight, as the whole Breadth of the Back to the perpendicular Height. WALLIS, KEIL, &c.   4. GRAVESANDE in his *Elements* (L. I. Chap. 13.) gives us the fame Proportion with the laft ; and in his *Scholium de ligno findendo*, tells us, that when the Parts of the Wood are feparated no farther than the Wedge is driven in, the Æquilibrium will be, when the Power is to the Refiftance, as half the Breadth of the Back of the Wedge to one of its Sides.

THOSE, who lay down the firft Proportion for determining the Power of this Machine, fuppofe the Parts, which are feparated from each other thereby, to recede from their firft Situation in Directions perpendicular to the Sides of the Wedge. Thus let ACB (*Fig.* 34.) reprefent a Wedge ; P, P, two Bodies to be feparated by it, the one to be moved towards I, the other towards F, in the Directions CI, and CF perpendicular to AC and CB ; then 'tis evident, that when the Wedge is driven in to the Situation MNO, the two Bodies will be moved to Q and Q ; that is, one will have

have paſſed through the Space CK, the other through CL, but theſe Spaces being equal, their Velocities are the ſame as if they had both paſſed over one of them, *v. g.* CL, or which is equal to it DG (drawn perpendicular to CB); therefore the Power, which we ſuppoſe applied at D, moves through DC, while the Obſtacle moves through DG, conſequently (§. 7.) when the Power is to the Weight as DG to DC, that is, as DB to CB *, or half the Back of the Wedge to one of its Sides, they will be in Æquilibrio. This Proportion therefore, when the Parts of the Weight are ſuppoſed to be moved by the Wedge in the Directions CI and CF, is true.

2. THE ſecond Proportion is alſo true, ſuppoſing the Bodies P, P, to recede from each other in the Directions CN and CM, parallel to AB the Back of the Wedge ; for, when the Wedge is driven in between them, to the Situation MNO, the Bodies will have moved through a Space as CN, or which is equal to it DB, half the Back of the Wedge, and the Power through a Space equal to its Height as before ; conſequently (§. 7.) in this Caſe, the Æquilibrium will be, when the Power is

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* For (8 *Elem.* 6.) the Triangles *DCG* and *DCB* are ſimilar and conſequently $DG : DC :: DB : CB$.

to

to the Weight, as half the Back of the Wedge to its Height *.

3. THOSE, who imagine there will not be an Æquilibrium, unless the Power be to the

* The same may be otherwife demonftrated from Section 14. Chapter 9. thus. Let there be a Body as *L* (*Fig.* 35.) drawn againft the Wedge *ABC* by the Weight *W*, in the Direction *LF*, parallel to the Back of the Wedge *AB*; but prevented from fliding down towards *C*, by a Plane (whofe upper Surface we may fuppofe reprefented by *EF*) lying under it. I fay, the Power will be to the Weight, when they are in Æquilibrio, as *DA* to *DC*.

Dem. The Body *L* is here acted upon in three Directions, *viz.* by the Force of the Weight *W* in the Direction *LF*, by the two Planes *CA* and *EF*, in the Directions *LG* and *LI*, perpendicular to their Surfaces; let *GE* be drawn parallel to *LI*, then will the Triangle *LGE* have all its Sides refpectively parallel to thofe Directions; confequently (Chap. 9. §. 14) if we fuppofe *LE* to exprefs the Force of the Weight *W*, *GL* will reprefent the Preffure of the Body *L* againft the Wedge; and if that is refolved into *GE* and *GH* the one perpendicular to the Direction of the Power, the other parallel and contrary to it; the laft, viz. *GE*, will exprefs the whole Force wherewith the Weight refifts the Motion of the Power; but *GE* is to *EL*, as *DA* to *DC* (for the Triangles *EGL* and *DAC* are fimilar, the Sides of one being *ex Conftruct.* refpectively perpendicular to thofe in the other; v. g. *LG* to *CA*, *EL* to *DC*, and *GE* to *DA*); confequently the Power is to the Weight, when they balance each other, as half the Breadth of the Back of the Wedge to its Height. *Q. E. D.*

Corol. Suppofe the Body *L* had been drawn againft the Wedge in the Direction *GL* perpendicular to its Surface, and were to be moved by it in the contrary Direction towards *G*, as in the firft Cafe; then if *GL* expreffes the Force with which it is drawn towards the Wedge, *GE* will be that with which it refifts the Power; but *GE* is to *GL* as *DA* to *AC*, the Triangles *EGL* and *DAC* being fimilar; confequently in this Cafe, the Power will be to the Weight, as half the Breadth of the Back of the Wedge to one of its Sides; as was before demonftrated.

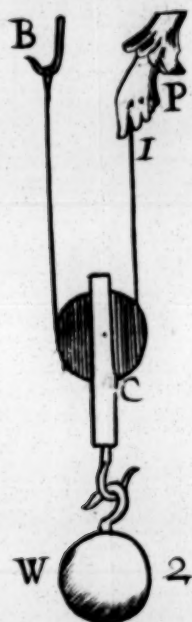Weight,

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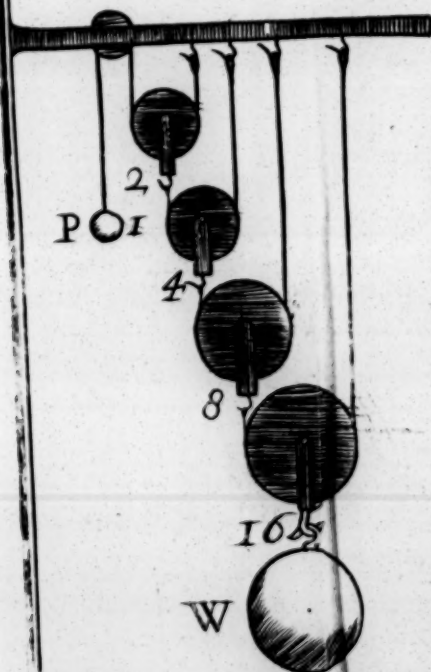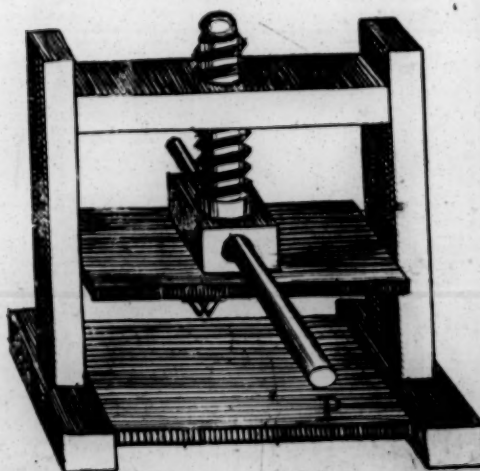rds
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Weight, as the whole Breadth of the Back of the Wedge to its Height; suppose, as in the last Case, that the Bodies to be separated recede from each other in Directions parallel to the Back of the Wedge; and endeavour to support their Opinion by the following Argument : *viz.* that, when the Wedge is driven in to the Situation MNO (*Fig.* 34.) as before, each Part of the Weight having moved through a Space equal to half the Back of the Wedge, the whole Weight has therefore moved through twice so much, or a Space equal to the whole Back : as much as to say, the Whole has moved farther than its Parts ; which is absurd.

4. THIS is GRAVESANDE's Mistake in his *Elements*, the same he has also made in his *Scholium de ligno findendo*, and thereby determined the Power in both Places to be twice as big, as it ought to be. If he had proceeded in the following Manner, his Argument would have been easier, as well as the Conclusion juster. Suppose the Wedge ABC driven into the Wood QLQ (as represented *Fig.* 36.) which is split no farther than the Point of the Wedge (or however no farther than is just sufficient to give it Room to move) which Case GRAVESANDE supposes in his *Scholium*, I say, that in this Situation of the Wedge, the Power is to the Weight, as one fourth Part of the Back of the Wedge to one of its Sides. For it is evident, that when the upper

Ends

Ends of the Wood, which prefs againft the Wedge in the Points G, H, are put into Motion by the Wedge, they will move in the Directions HI and GF, perpendicular to the Sides of the Wedge, becaufe they turn as it were upon a Joint at L, which we fuppofe contiguous to C : again, fince only the upper Ends of the Wood are put into Motion, and not the lower ones, which remain at L ; 'tis evident, that the Motion of each Piece (fuppofing their Thicknefs the fame from End to End, and their Subftance uniform) will be but half, what it would have been had the lower ones moved with the fame Degree of Velocity. Now, were all the Parts of the Wood to have the fame Degree of Velocity, the Power would be to the Weight, as in the firft Cafe, *viz.* as DB to BC (*Fig.* 34.) ; therefore in this Cafe, it is as half DB to BC, or as one fourth Part of the Back of the Wedge to one of its Sides. Which was to be proved.

XX. The Form of the *Inclined Plane* being no other than that of half a Wedge, as is manifeft from the Reprefentation of it (*Fig.* 37.) it follows that what has been demonftrated of the one, may be applied to the other, and the Properties of both will be found the fame. For Inftance, if the Weight W is to be raifed up the Plane CB, by the Power P, in a Direction parallel to the Plane ; inftead of that, we may fuppofe the Weight prevented from
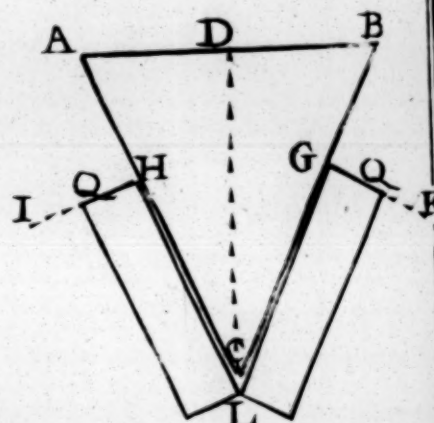
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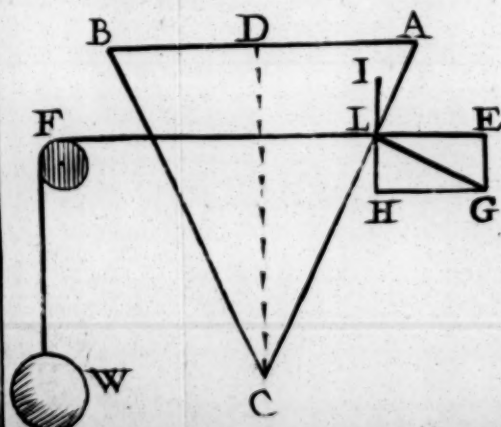ma-
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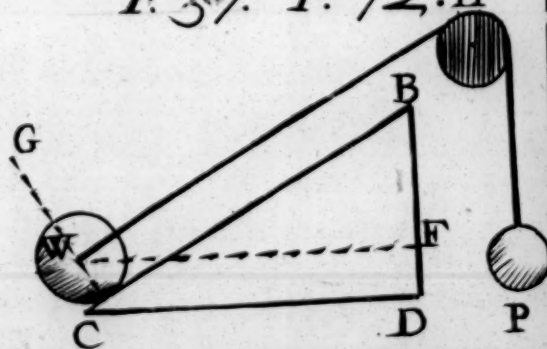and
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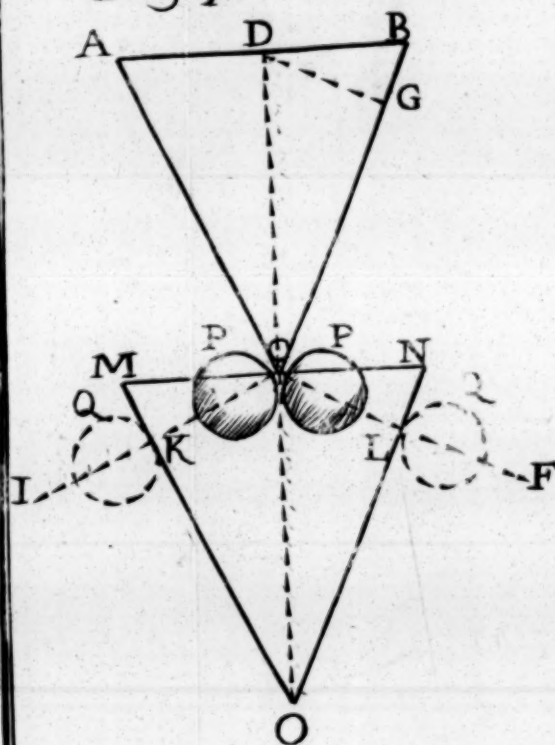
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running off the Plane by the String WB, and the inclined Plane driven under it like a Wedge in the Direction DC : then will the Weight rife towards G in a Direction perpendicular to CB, for we muft always fuppofe the String WB parallel to the Plane (as it would have been, if the Weight had been drawn up by it ;) then will the Action of the Plane upon the Weight be fimilar to that of the Wedge in the firft Cafe : and confequently the Power will bear fuch Proportion to the Weight, as DB to BC; that is, as the Height of the Plane to its Length.   Again, fuppofe the Weight was to have been drawn up the Plane by a String in the Direction WF parallel to CD the Bafe of the inclined Plane ; then if the Plane be driven under the Weight as before, it muft rife in a Direction perpendicular to CD, that is, parallel to DB : then the Cafe will be analagous to the fecond of the Wedge ; confequently, the Power will be to the Refiftance of the Weight, when there is an Æquilibrium, in the Proportion of DB to DC, as there demonftrated.

XXI. These are the Powers or Machines, which, under different Forms, conftitute all other how complicated foever ; and as the Æquilibrium in any one of thefe is, when the Power and Weight are inverfely as their Velocities ; fo in a Machine however compounded, the Power and the Weight will exactly

K   balance

balance each other, when they are in this Proportion ; for by §. 7. their Moments will then be equal, and the Machine, if at reft, will continue in that State ; and, if put into Motion by an external Force, will gradually lofe it, when that Force ceafes to act ; on account of the unavoidable Friction of the Machine, and the Refiftance of the Air, which it muft neceffarily meet with, unlefs its Motion could be performed in a perfect Vacuum. From hence we fee the Impoffibility of contriving an Engine, whofe Motion fhould be *perpetual,* that is, fuch as does not owe its Continuance to the Application of fome external Force; a Problem that has given Birth to an almoft infinite Number of Schemes and Contrivances. For unlefs fome Method could be found out of gaining a Force, by the artful Difpofition and Combination of the Mechanical Powers, equivalent to that which is continually deftroyed by Friction, and the Refiftance of the Air, the Motion, which was at firft given to the Machine, muft at length be necefiarily loft. But we fee, that thofe Inftruments are only different Means, whereby one Body communicates its Motion to another; and not defigned to produce a Force which had no Exiftence before. 'Tis for want of a due Confideration of this, that fo many Mechanical Defigns have proved abortive, fo many Engines unequal to the Performance for which they

they were defigned, and fo many Impoſſibi-
lities attempted.

" If it were poſſible, ſays Bp. WILKINS,
" to contrive ſuch an Invention, whereby
" any conceivable Weight may be moved by
" any conceivable Power, both with equal
" Velocity (as it is in thoſe Things which are
" immediately ſtirred by the Hand, without
" the Help of any other Inſtrument) the
" Works of Nature would be then too much
" ſubjected to the Power of Art ; and Men
" might be thereby encouraged (with the
" Builders of *Babel*, or the Rebel Giants) to
" ſuch bold Deſigns, as would not become
" a created Being.   And therefore the Wiſdom
" of Providence has ſo confined theſe Human
" Arts, that what an Invention hath, in the
" *Strength* of its Motion, is abated in the *Slow-*
" *neſs* of it ; and what it has, in the extraordi-
" nary *Quickneſs* of its Motion, muſt be al-
" lowed for in the great *Strength* requiſite in
" the Power which is to move it *.

* *Wilkins*'s Mathem. Magick. p. 104.